Pavage apériodique du plan

La quête de la tuile unique

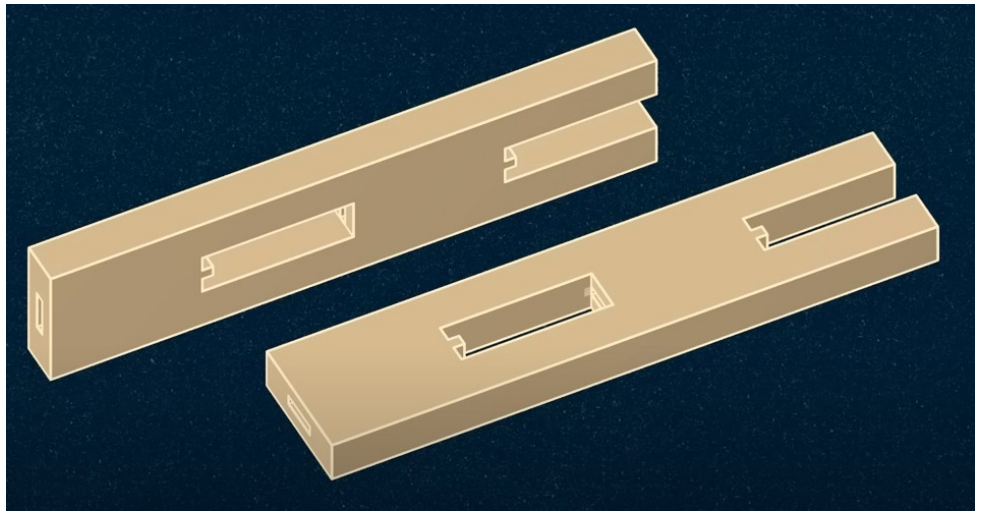
18e problème de Hilbert

- (b) Existe-t-il [...] des polyèdres qui ne se présentent pas comme régions fondamentales de groupes de déplacements, et au moyen desquels cependant on peut, en juxtaposant convenablement les exemplaires congruents, arriver à remplir sans lacune l'espace tout entier?

Reinhart, *Zur Zerlegung der euklidischen Räume in kongruente Polytope* (1928)
Heesch, Aufbau der Ebene aus kongruenten Bereichen (1935)

18e problème de Hilbert

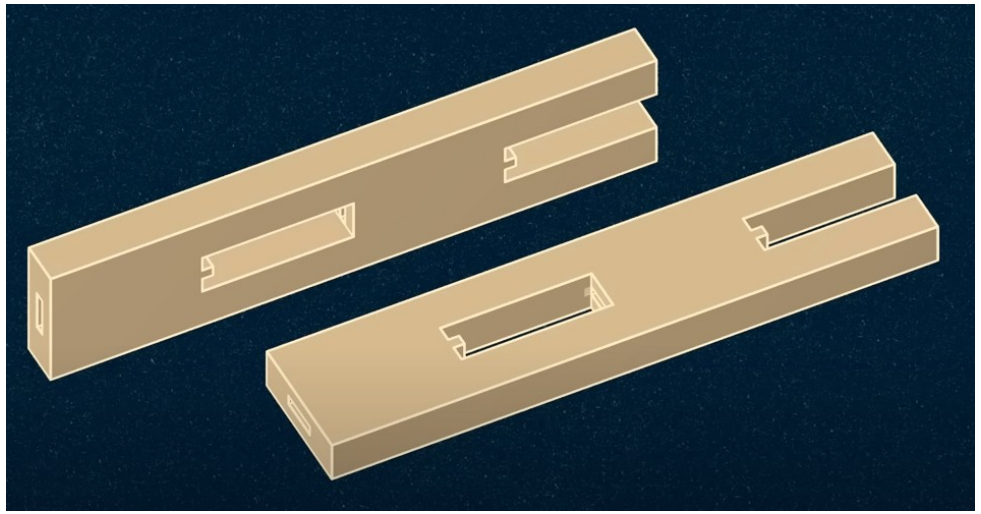
- (b) Existe-t-il [...] des polyèdres qui ne se présentent pas comme régions fondamentales de groupes de déplacements, et au moyen desquels cependant on peut, en juxtaposant convenablement les exemplaires congruents, arriver à remplir sans lacune l'espace tout entier?

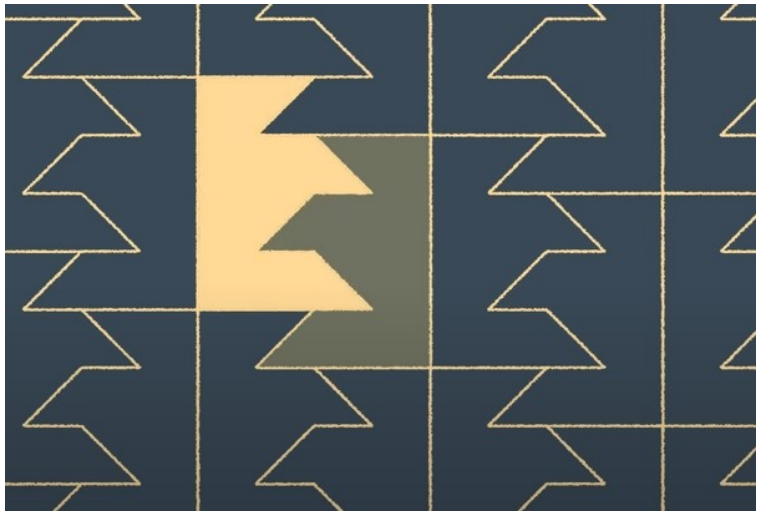


Reinhart, *Zur Zerlegung der euklidischen Räume in kongruente Polytope* (1928)
Heesch, Aufbau der Ebene aus kongruenten Bereichen (1935)

18e problème de Hilbert

- (b) Existe-t-il [...] des polyèdres qui ne se présentent pas comme régions fondamentales de groupes de déplacements, et au moyen desquels cependant on peut, en juxtaposant convenablement les exemplaires congruents, arriver à remplir sans lacune l'espace tout entier?



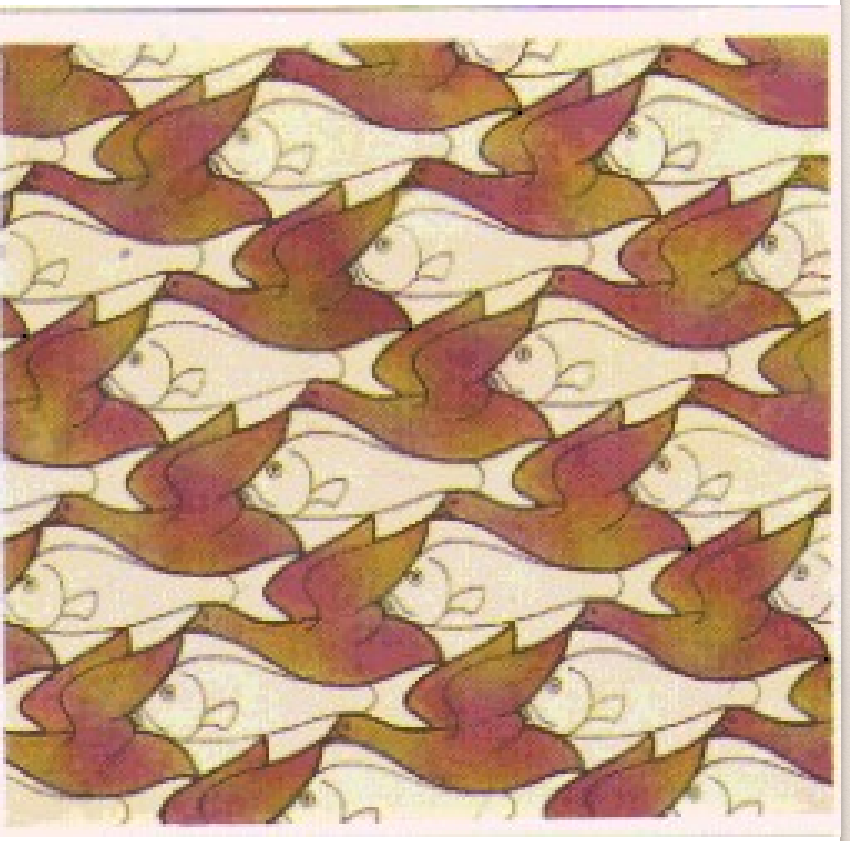


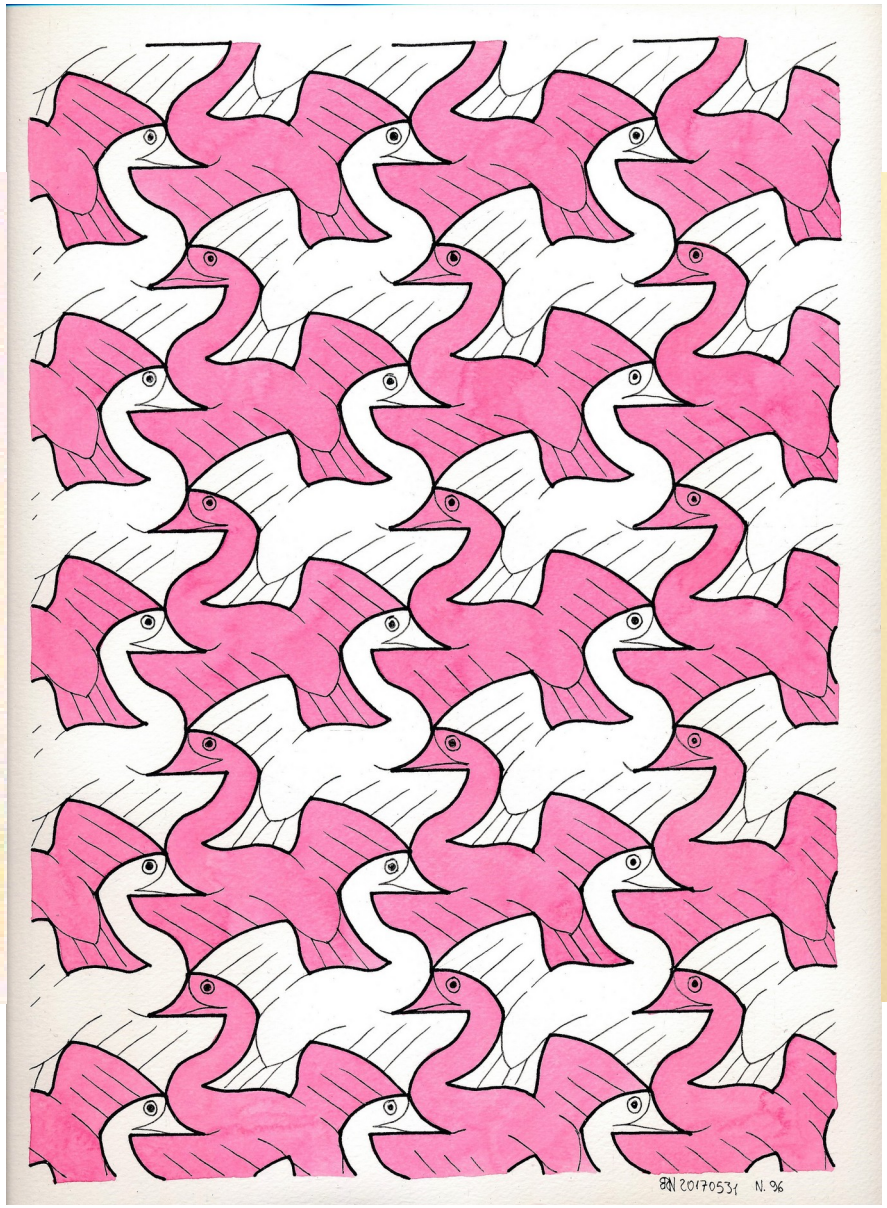
Reinhart, *Zur Zerlegung der euklidischen Räume in kongruente Polytope* (1928)
Heesch, Aufbau der Ebene aus kongruenten Bereichen (1935)

Pavage du plan

- “Remplir l’espace de formes géométriques (tuiles) telles que les formes ne se superposent pas et qu’il ne reste pas de trous”

C’est joli

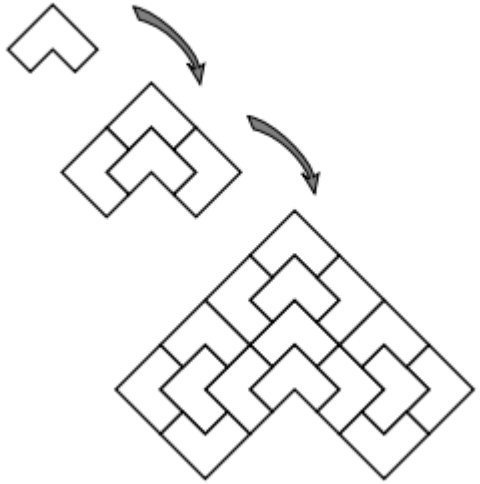


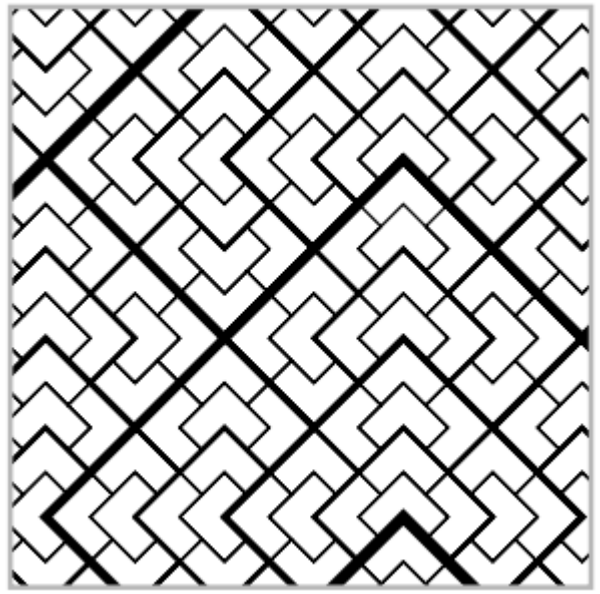


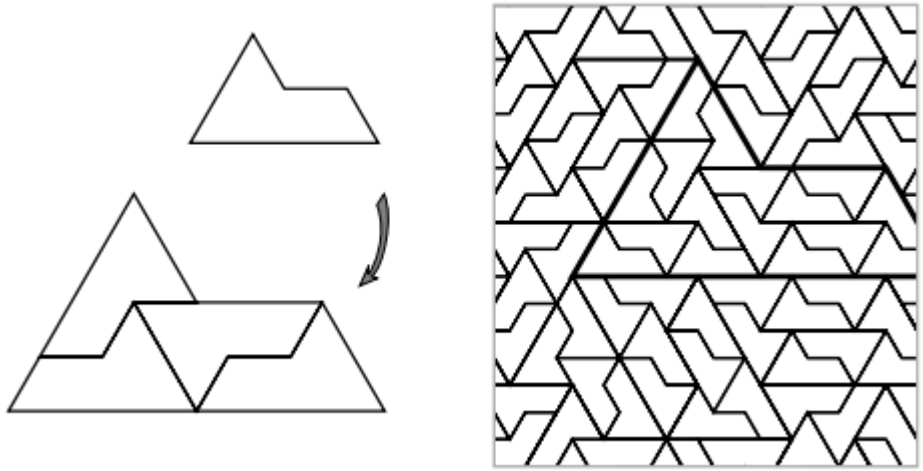
MC. Escher

Apériodicité

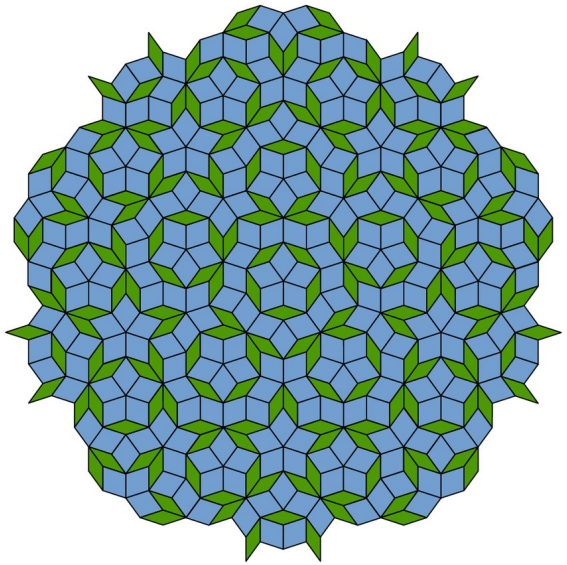
- Retirer la possibilité de retrouver une copie de ce pavage après une translation





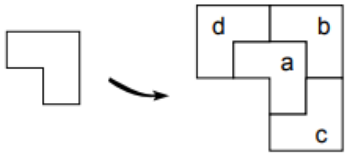


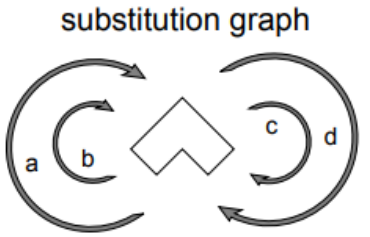
Exemple de rep-tiles

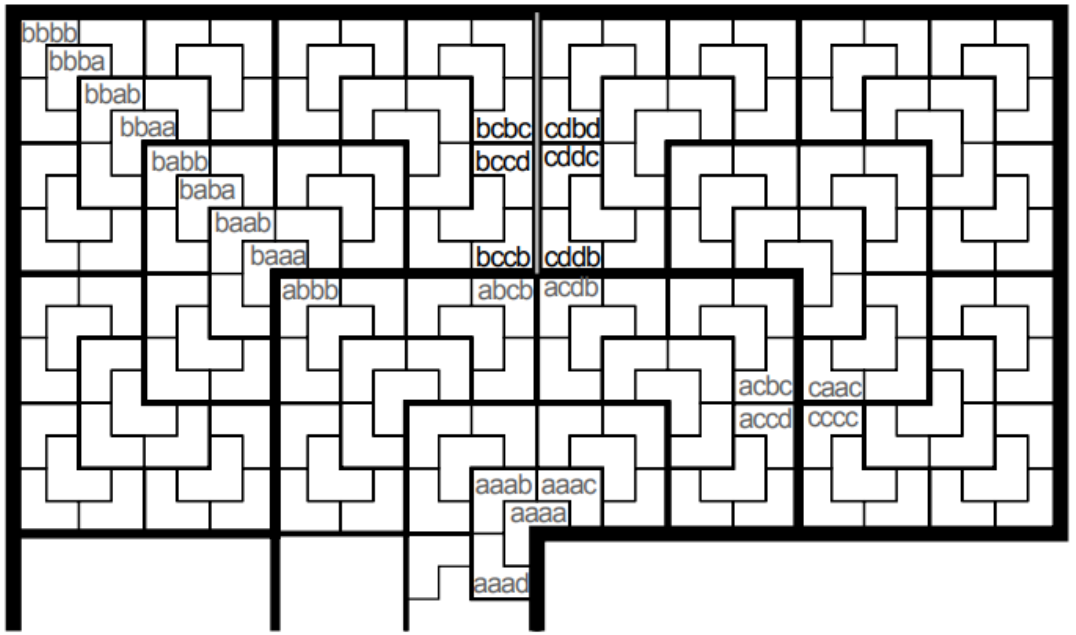


Preuve par la méthode de hiérarchie unique

- Les motifs se répètent récursivement.
- On peut donner des “coordonnées” dans la hierarchie
- Deux tuiles différentes ont des coordonnées différentes,
- => Donc pas de répétition



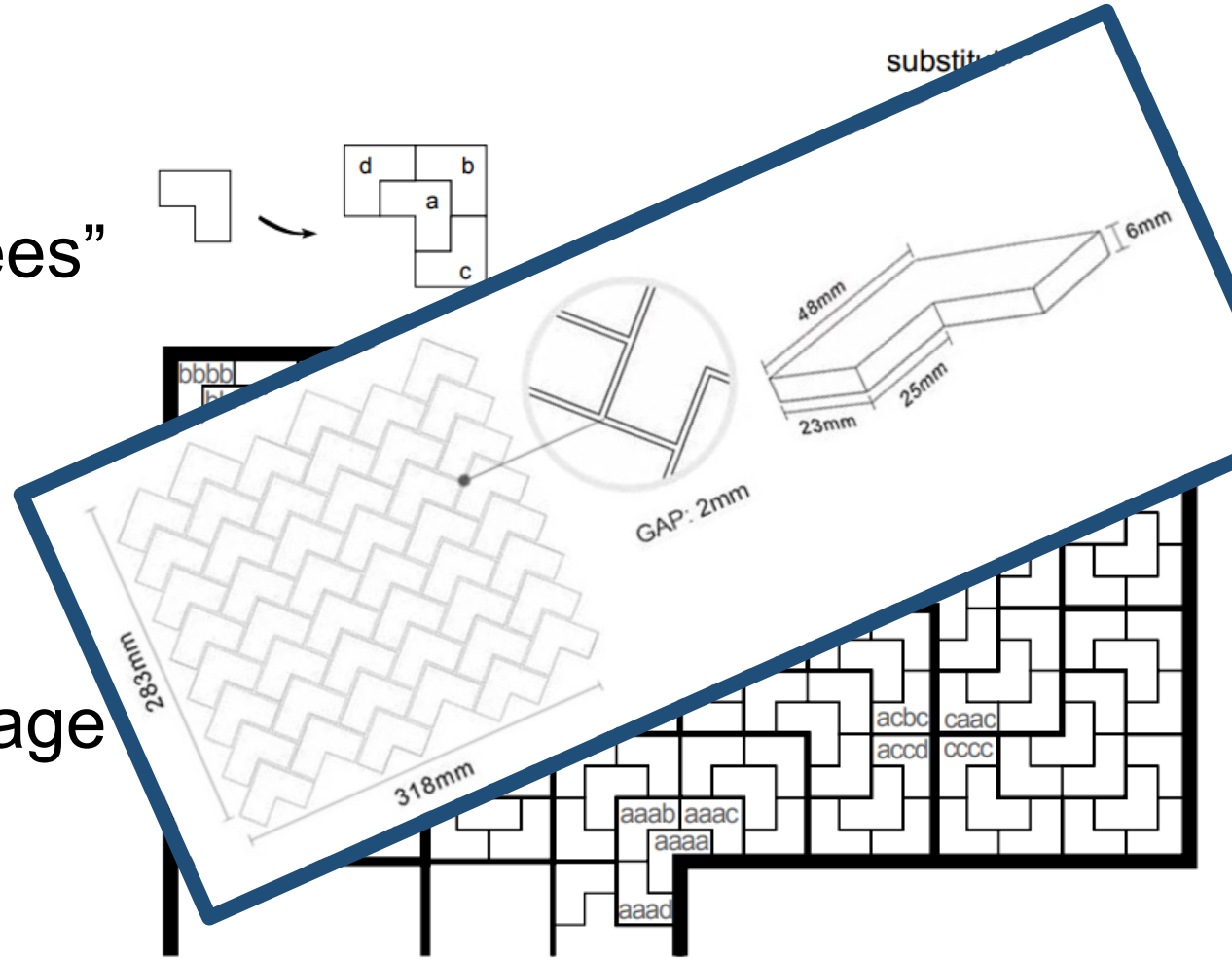




Goodman-Strauss, *Aperiodic hierarchical tiling* (1999)

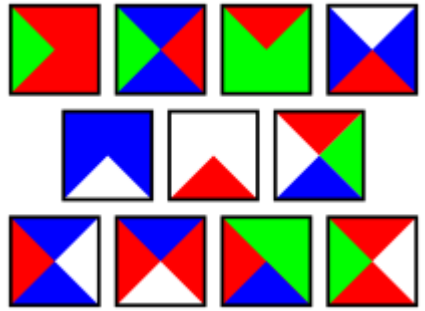
Preuve par la méthode de hiérarchie unique

- Les motifs se répètent récursivement.
- On peut donner des “coordonnées” dans la hierarchie
- Deux tuiles différentes ont des coordonnées différentes,
- => Donc pas de répétition
- Mais ici, il y a possibilité de pavage périodique (preuve par Ikea)



Goodman-Strauss, *Aperiodic hierarchical tiling* (1999)

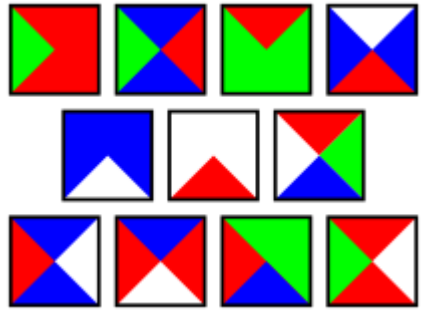
Réduire le nombre de tuiles



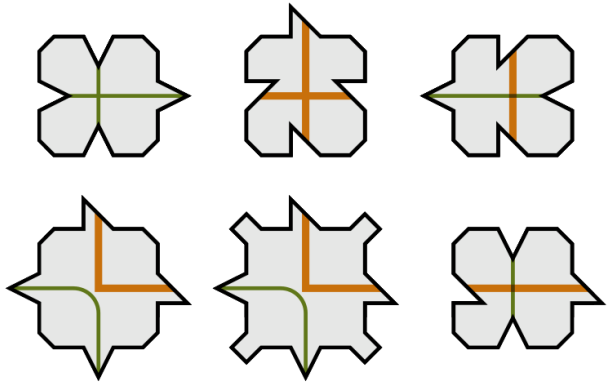
- Berger (1964) propose un système de 20 426 tuiles
- Jeandel et Roa (2015) propose 11 tuiles
- Robinson (1969) propose 6 tuiles
- Penrose (1979) trouve 2 tuiles
- Socolar-Taylor (2011) a 1 tuile!

Berger, *The Undecidability of the Domino Problem* (1964)
Jeandel and Rao, *An aperiodic set of 11 Wang tiles* (2015)
Robinson, *Undecidability and Nonperiodicity for Tilings of the Plane* (1971)
Penrose, *Pentaplexity: A class of non-periodic tilings of the plane* (1979)
Soloral and Taylor, *An aperiodic hexagonal tile* (2011)

Réduire le nombre de tuiles

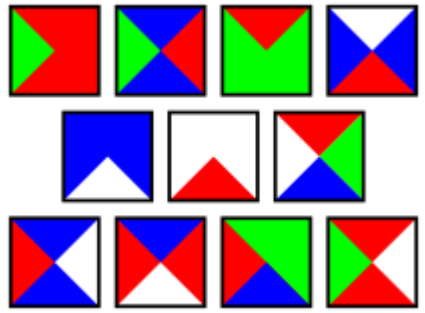


- Berger (1964) propose un système de 20 426 tuiles
- Jeandel et Roa (2015) propose 11 tuiles
- Robinson (1969) propose 6 tuiles
- Penrose (1979) trouve 2 tuiles
- Socolar-Taylor (2011) a 1 tuile!

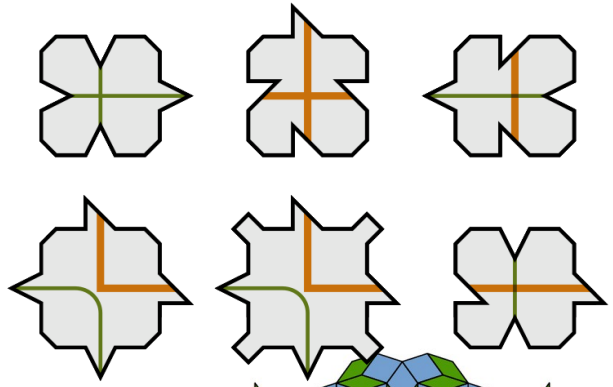


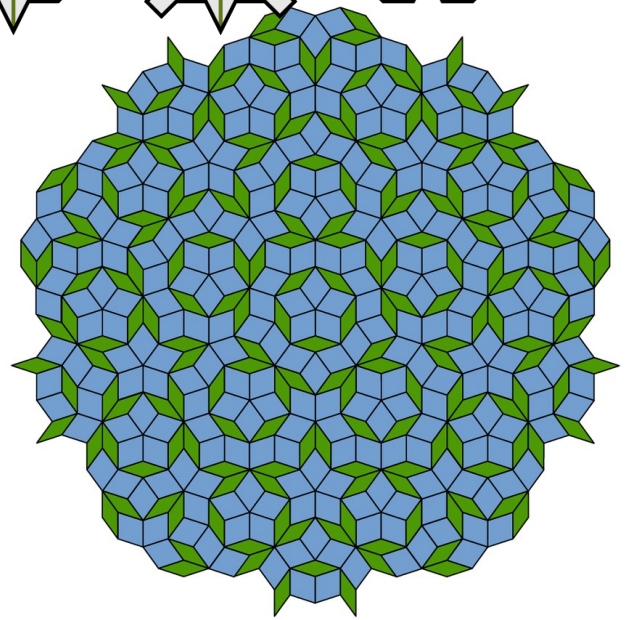
Berger, *The Undecidability of the Domino Problem* (1964)
Jeandel and Rao, *An aperiodic set of 11 Wang tiles* (2015)
Robinson, *Undecidability and Nonperiodicity for Tilings of the Plane* (1971)
Penrose, *Pentaplexity: A class of non-periodic tilings of the plane* (1979)
Soloral and Taylor, *An aperiodic hexagonal tile* (2011)

Réduire le nombre de tuiles



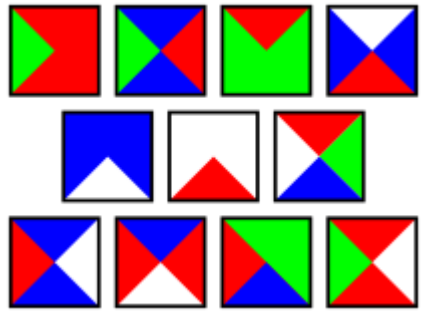
- Berger (1964) propose un système de 20 426 tuiles
- Jeandel et Roa (2015) propose 11 tuiles
- Robinson (1969) propose 6 tuiles
- Penrose (1979) trouve 2 tuiles
- Socolar-Taylor (2011) a 1 tuile!



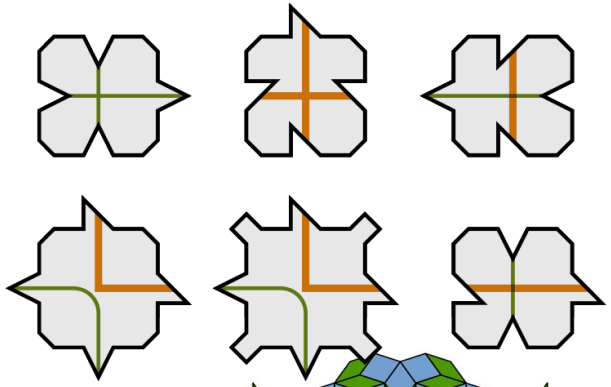


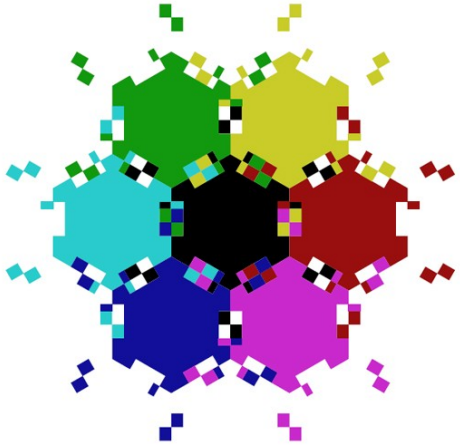
Berger, *The Undecidability of the Domino Problem* (1964)
Jeandel and Rao, *An aperiodic set of 11 Wang tiles* (2015)
Robinson, *Undecidability and Nonperiodicity for Tilings of the Plane* (1971)
Penrose, *Pentaplexity: A class of non-periodic tilings of the plane* (1979)
Soloral and Taylor, *An aperiodic hexagonal tile* (2011)

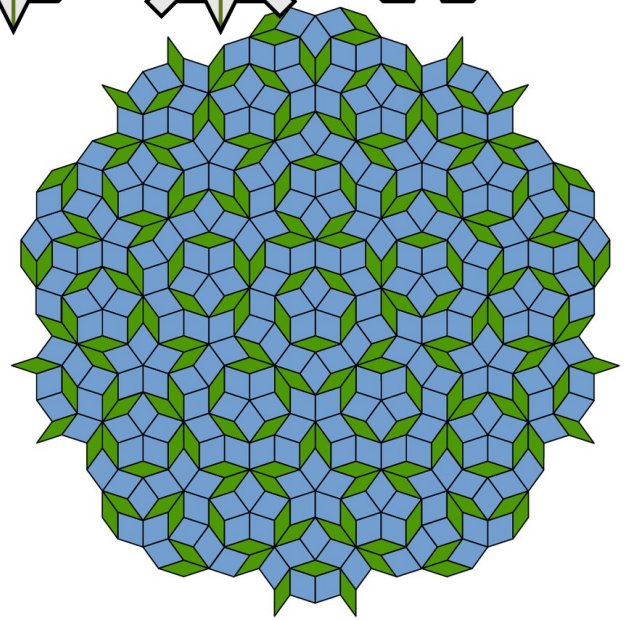
Réduire le nombre de tuiles



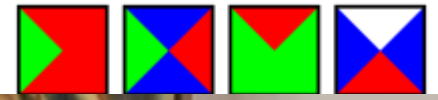
- Berger (1964) propose un système de 20 426 tuiles
- Jeandel et Roa (2015) propose 11 tuiles
- Robinson (1969) propose 6 tuiles
- Penrose (1979) trouve 2 tuiles
- Socolar-Taylor (2011) a 1 tuile!





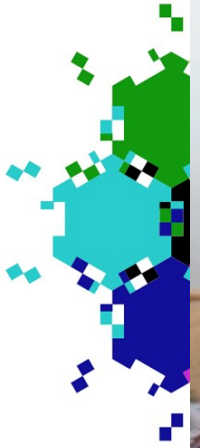


Berger, *The Undecidability of the Domino Problem* (1964)
Jeandel and Rao, *An aperiodic set of 11 Wang tiles* (2015)
Robinson, *Undecidability and Nonperiodicity for Tilings of the Plane* (1971)
Penrose, *Pentaplexity: A class of non-periodic tilings of the plane* (1979)
Soloral and Taylor, *An aperiodic hexagonal tile* (2011)



Réduire le nombre de tuile

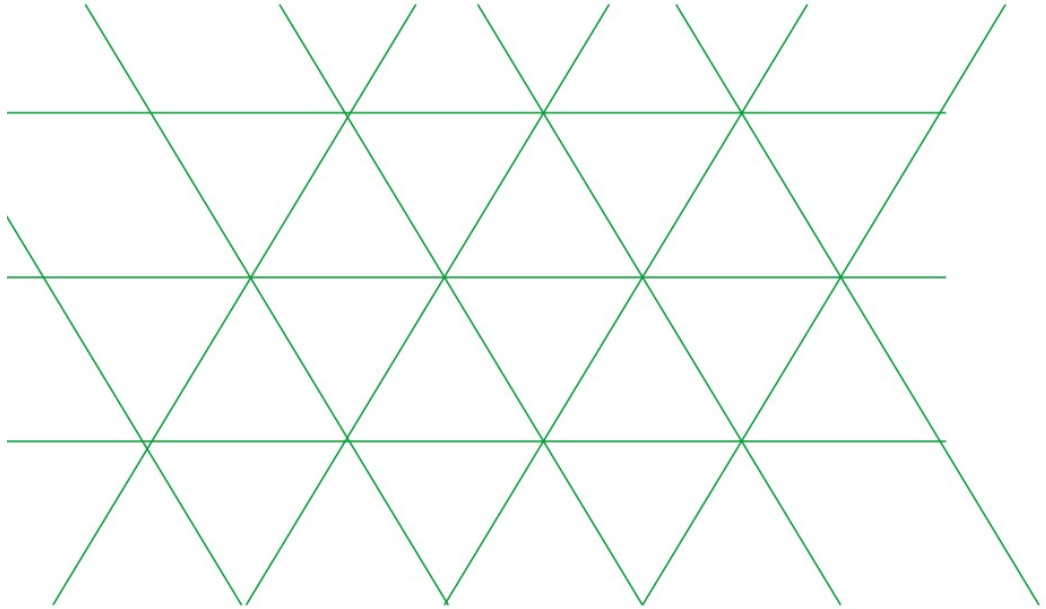
- Berger (1964) propose un système de
- Jeandel et Roa (2015) propose 11 tuile
- Robinson (1969) propose 6 tuiles
- Penrose (1979) trouve 2 tuiles
- Socolar-Taylor (2011) a 1 tuile!



Berger, *The Undecidability of the Domino Problem* (1964)
Jeandel and Rao, *An aperiodic set of 11 Wang tiles* (2015)
Robinson, *Undecidability and Nonperiodicity for Tilings of the Plane* (1
Penrose, *Pentaplexity: A class of non-periodic tilings of the plane* (197
Soloral and Taylor, *An aperiodic hexagonal tile* (2011)

Le “Chapeau”

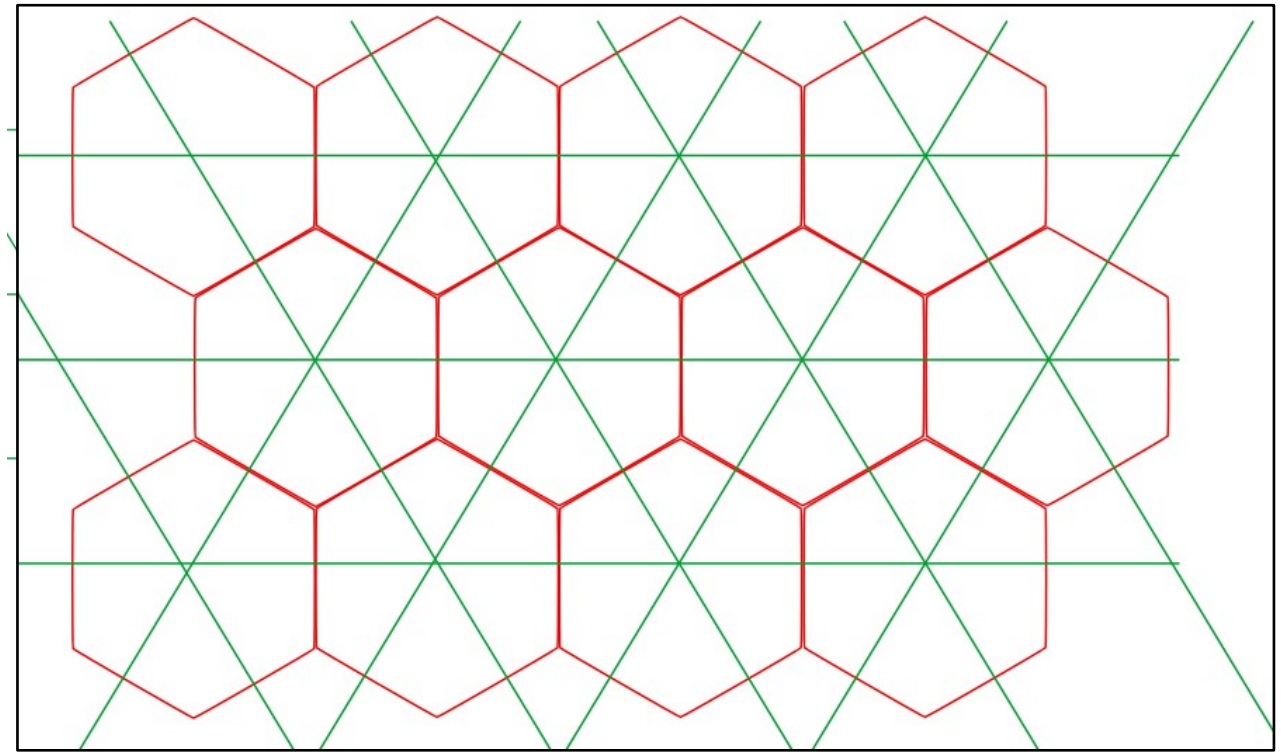
- Construction facile



Smith, Myers, Kaplan and Goodman-Strauss, *An aperiodic monotile* (2023)

Le “Chapeau”

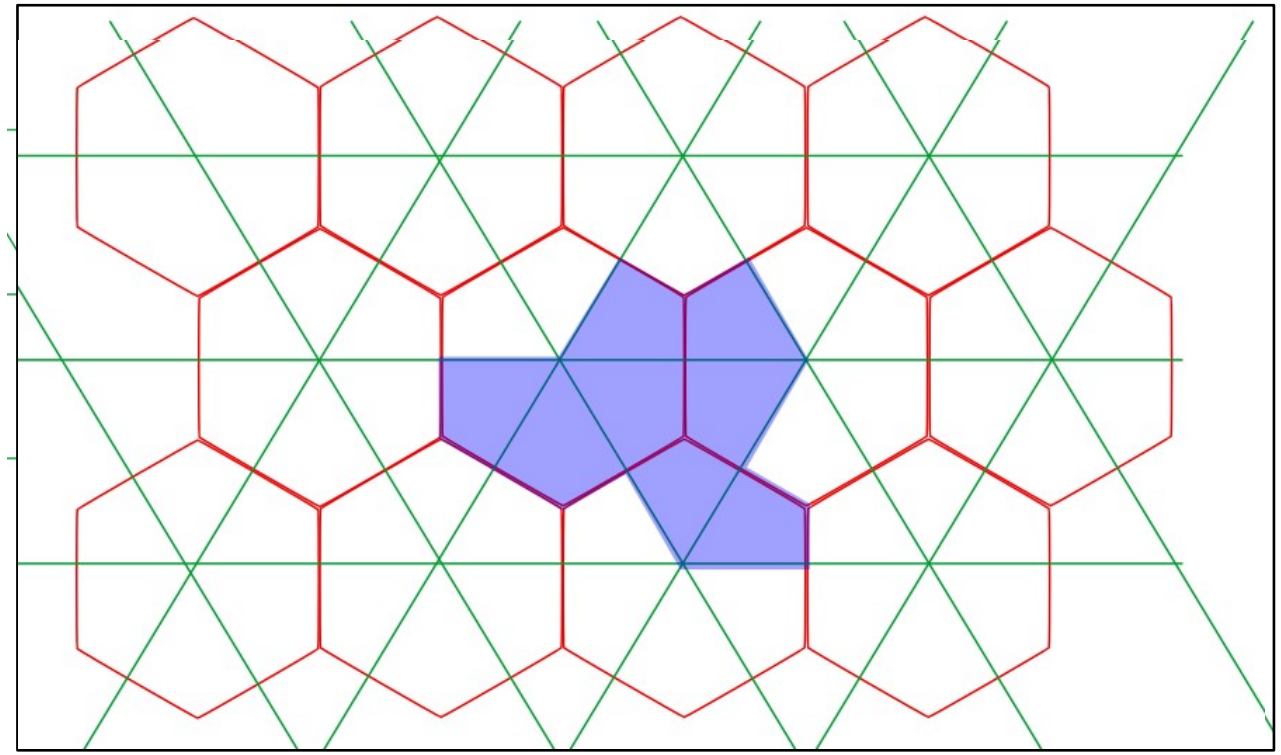
- Construction facile



Smith, Myers, Kaplan and Goodman-Strauss, *An aperiodic monotile* (2023)

Le “Chapeau”

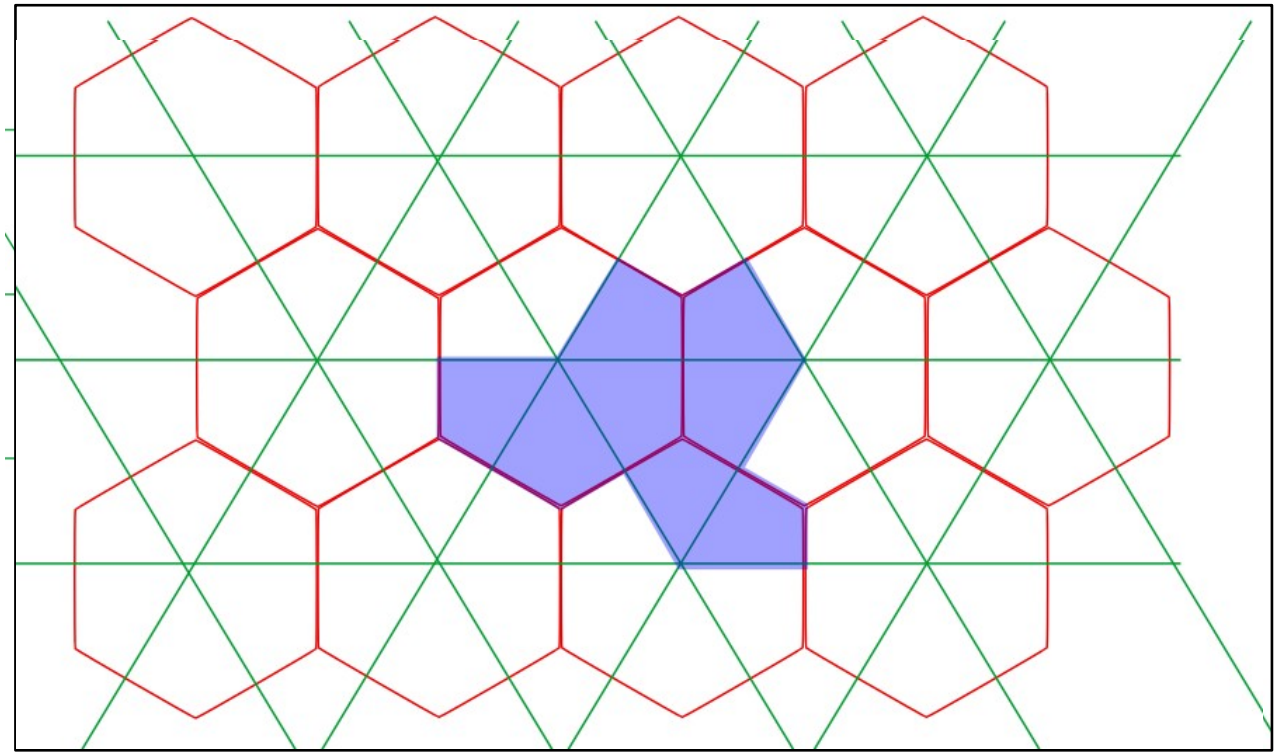
- Construction facile



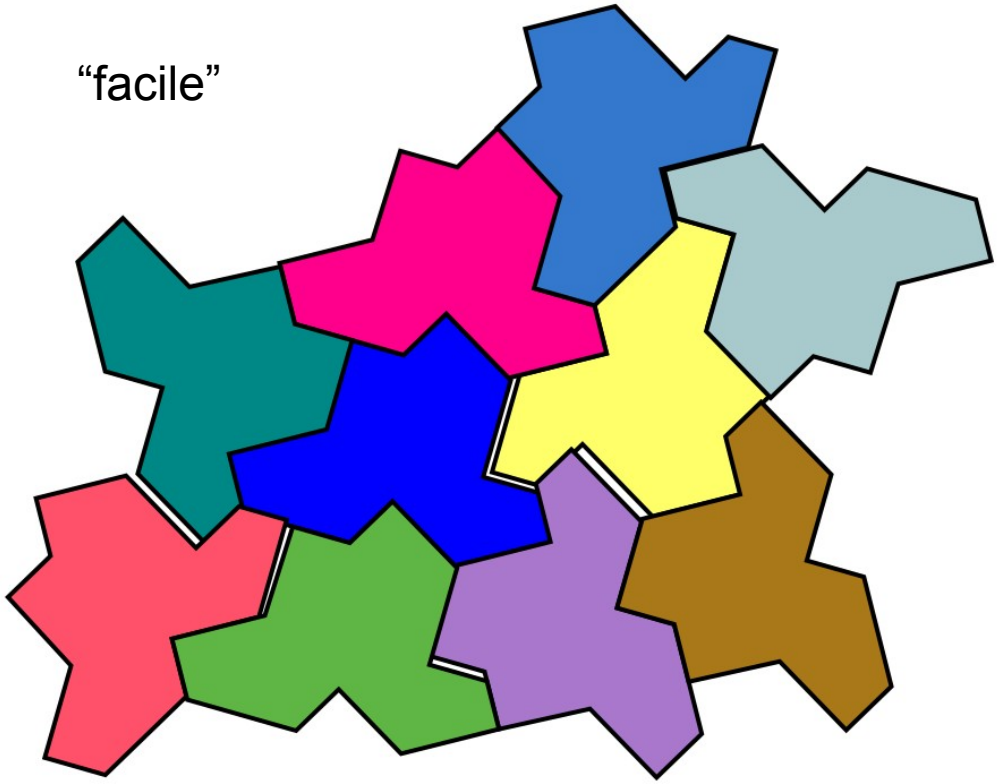
Smith, Myers, Kaplan and Goodman-Strauss, *An aperiodic monotile* (2023)

Le “Chapeau”

- Construction facile



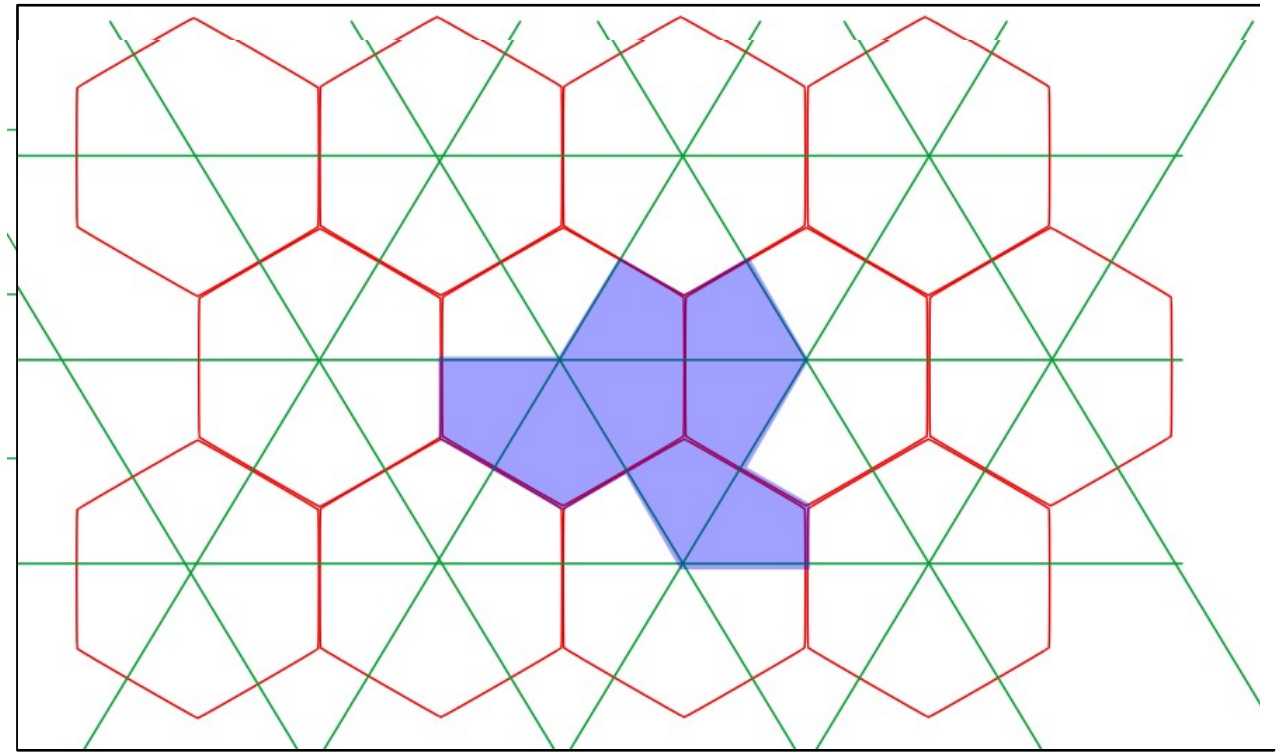
“facile”

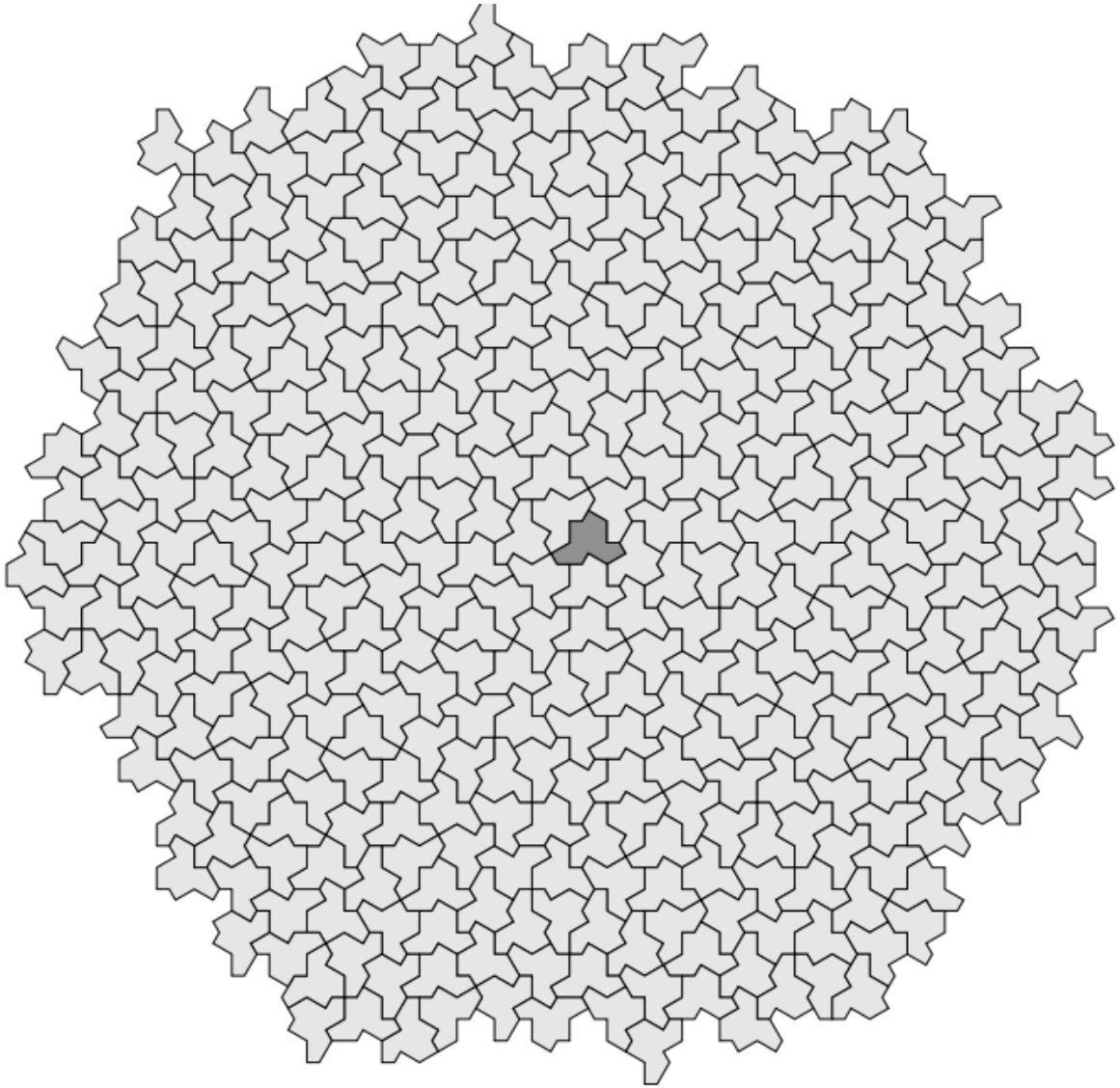


Smith, Myers, Kaplan and Goodman-Strauss, *An aperiodic monotile* (2023)

Le “Chapeau”

- Construction facile

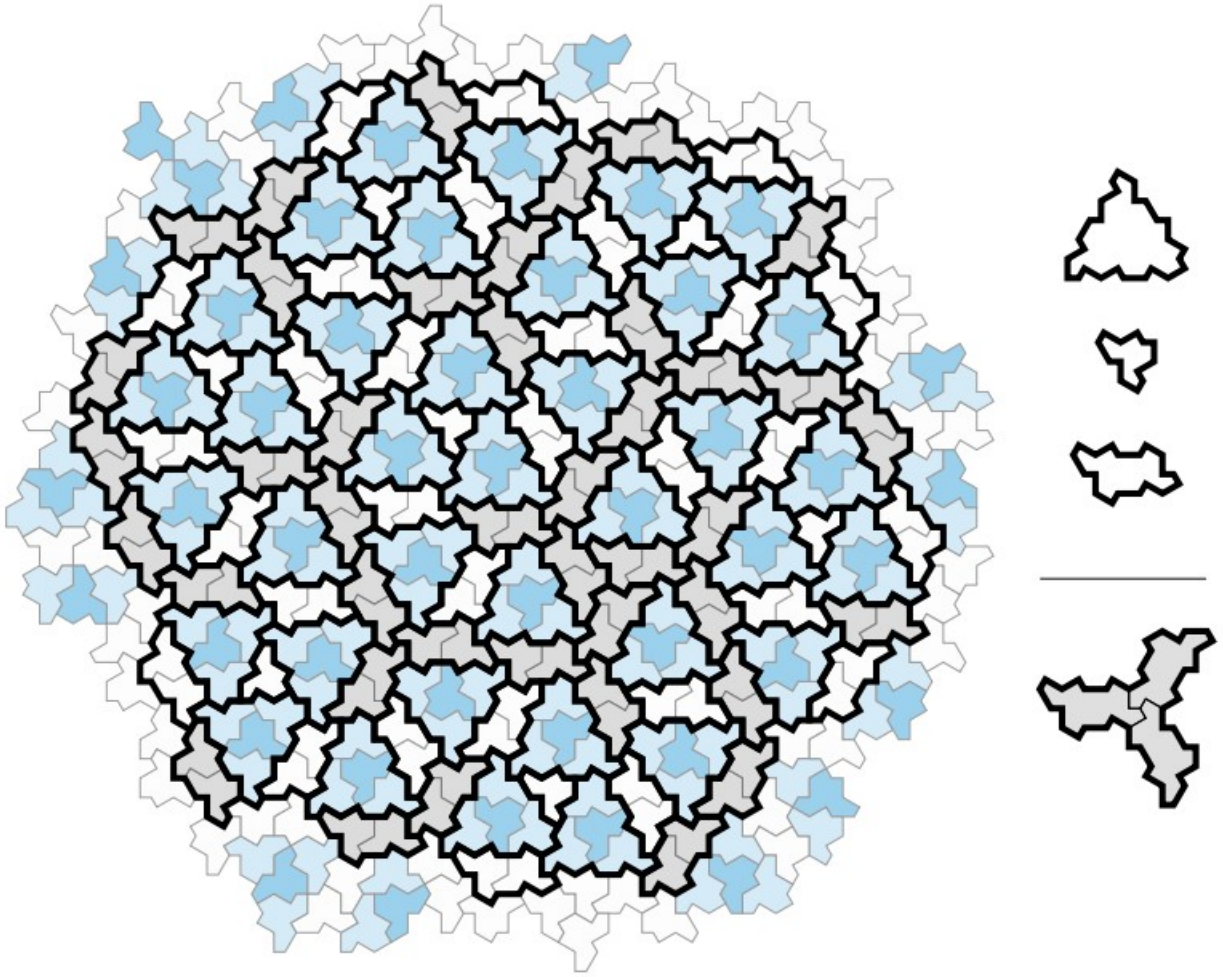




Smith, Myers, Kaplan and Goodman-Strauss, *An aperiodic monotile* (2023)

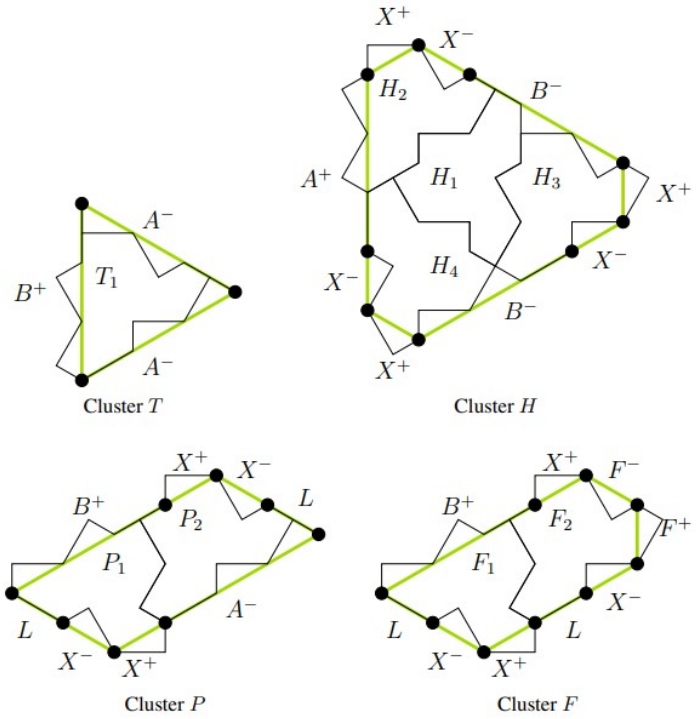
Apériodique?

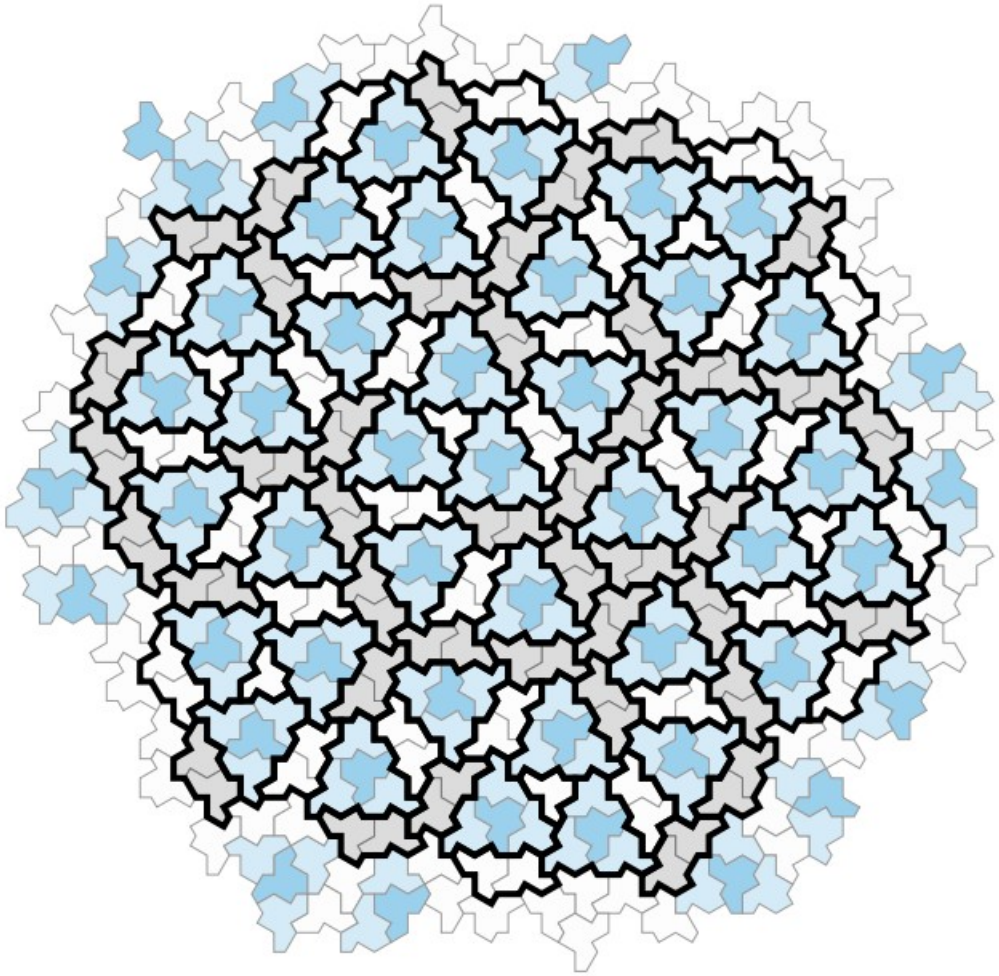
- Trouver des “clusters”
- Remplacer les clusters par des “meta-tiles”



Apériodique?

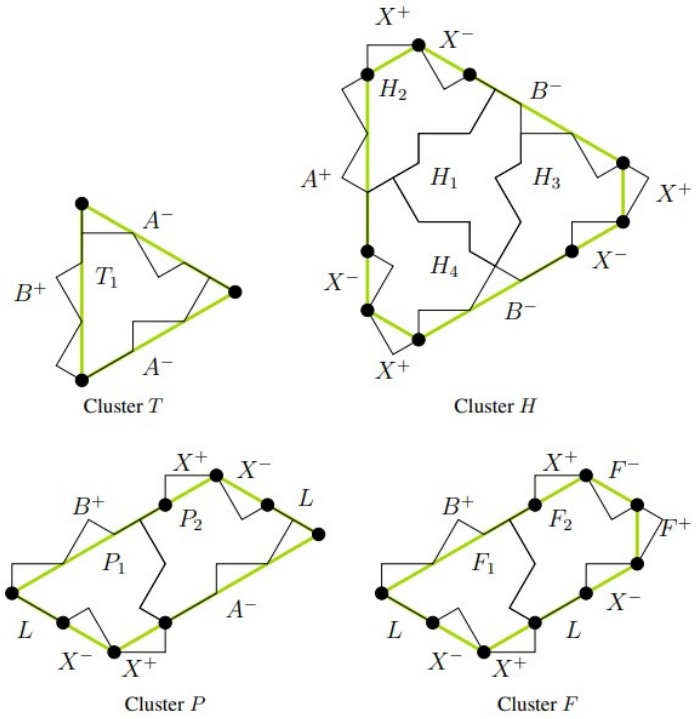
- Trouver des “clusters”
- Remplacer les clusters par des “meta-tiles”

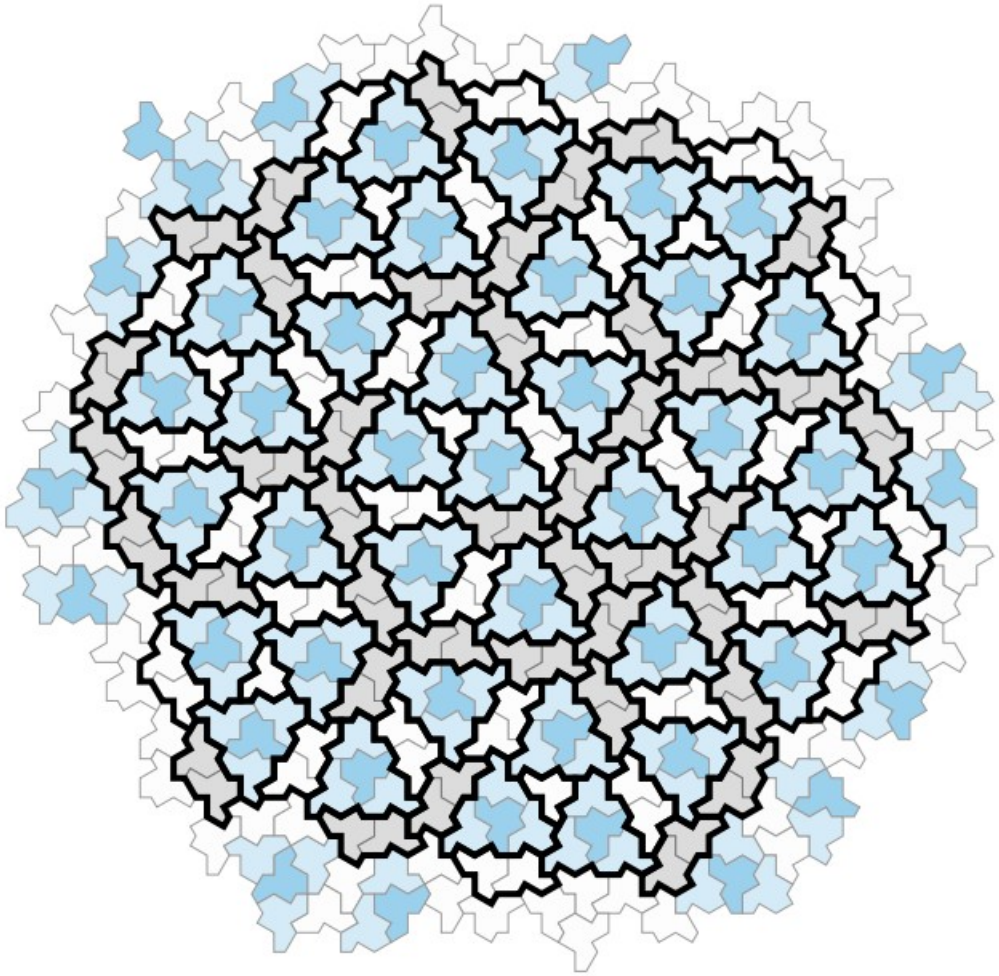


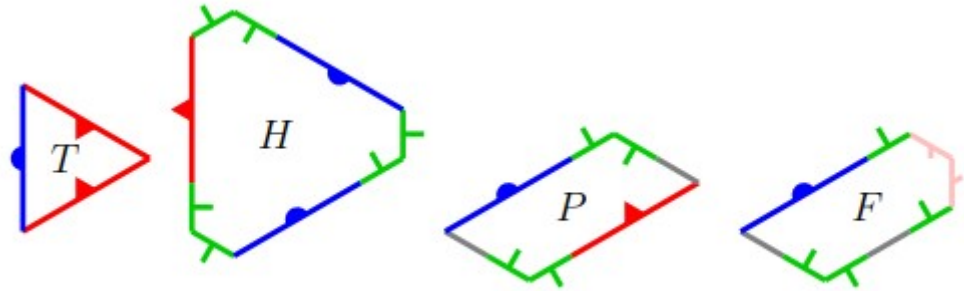


Apériodique?

- Trouver des “clusters”
- Remplacer les clusters par des “meta-tiles”

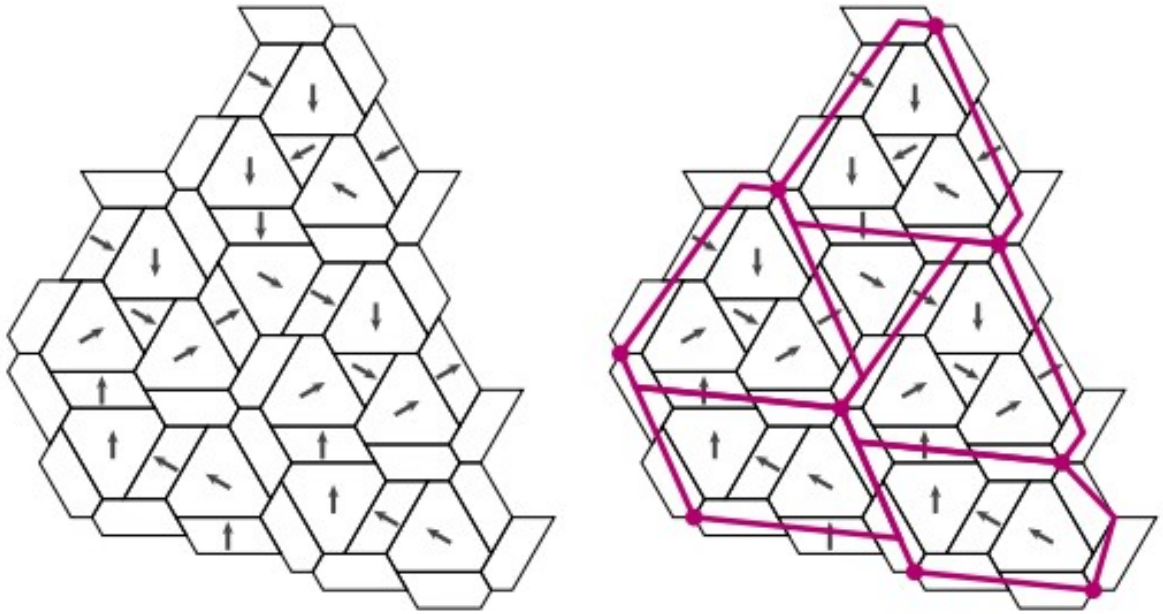






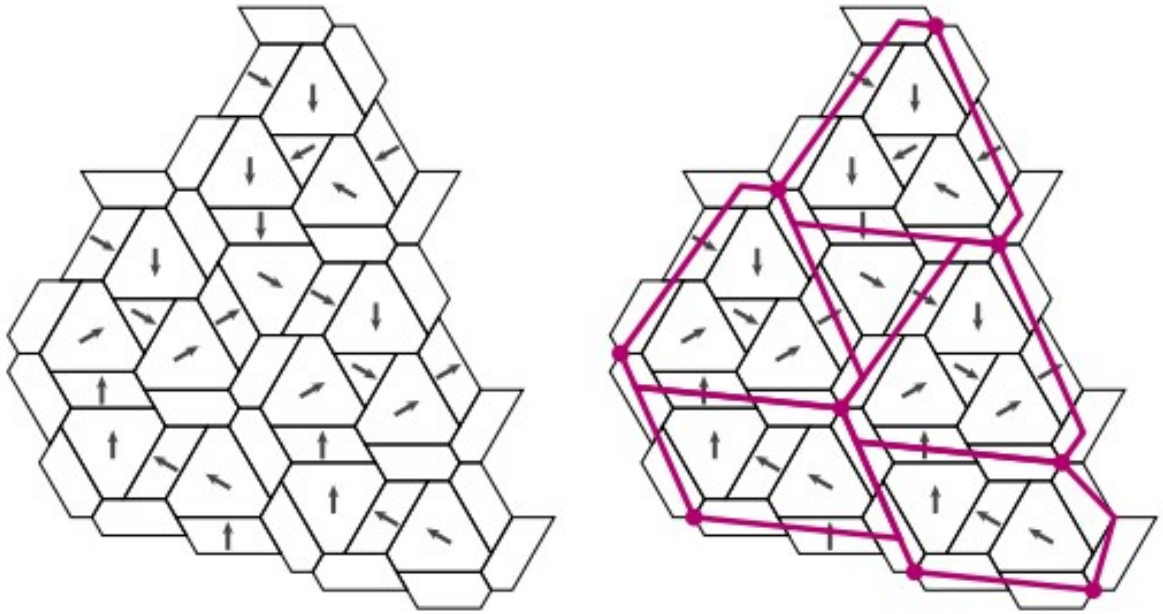
Apériodique?

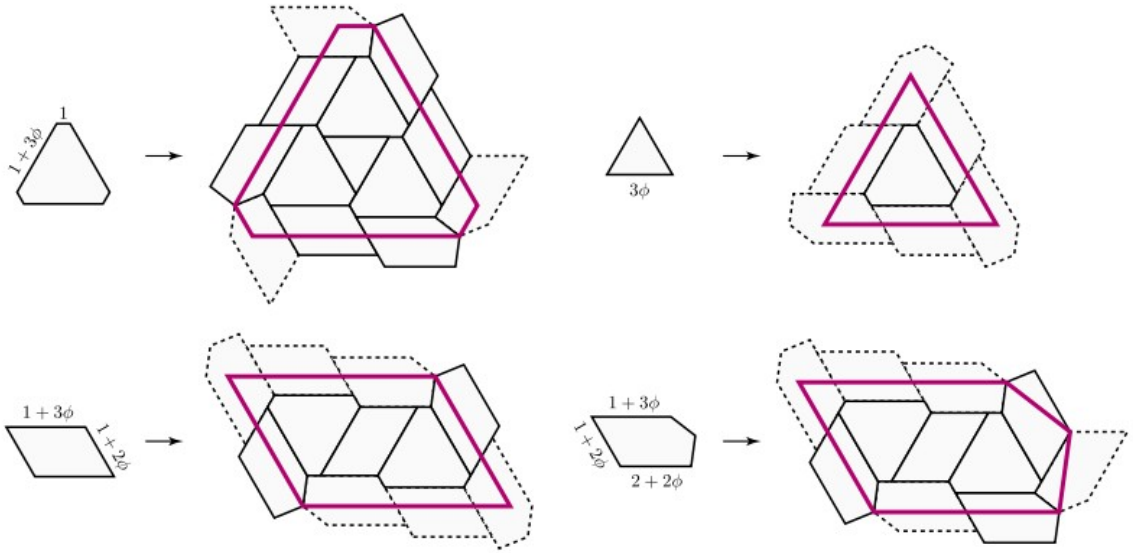
- Paver l’espace de meta-tiles
- Trouver des “clusters”
- Remplacer les clusters par des “meta-meta-tiles”



Apériodique?

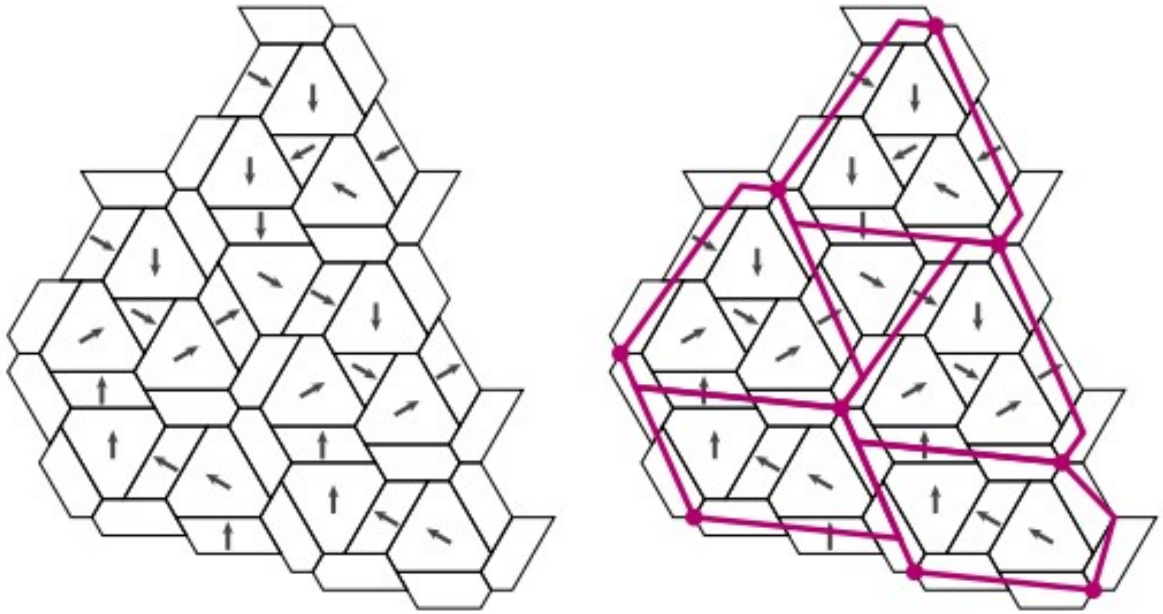
- Paver l’espace de meta-tiles
- Trouver des “clusters”
- Remplacer les clusters par des “meta-meta-tiles”

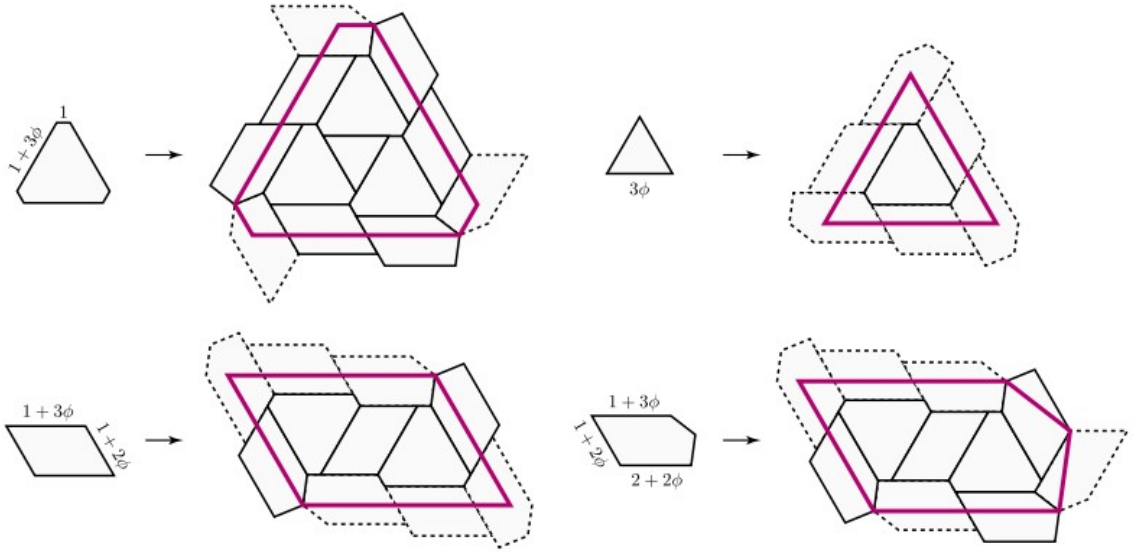


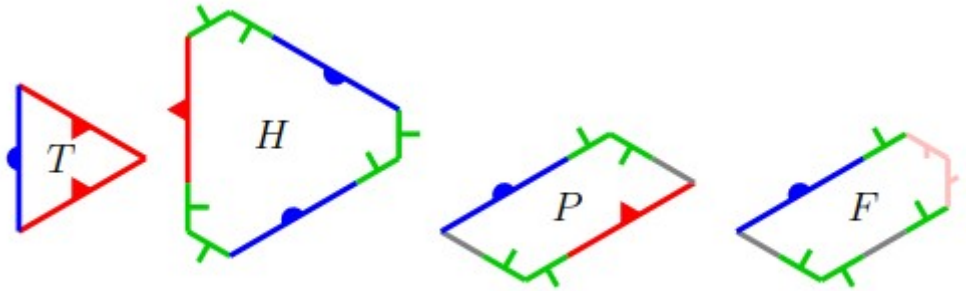


Apériodique?

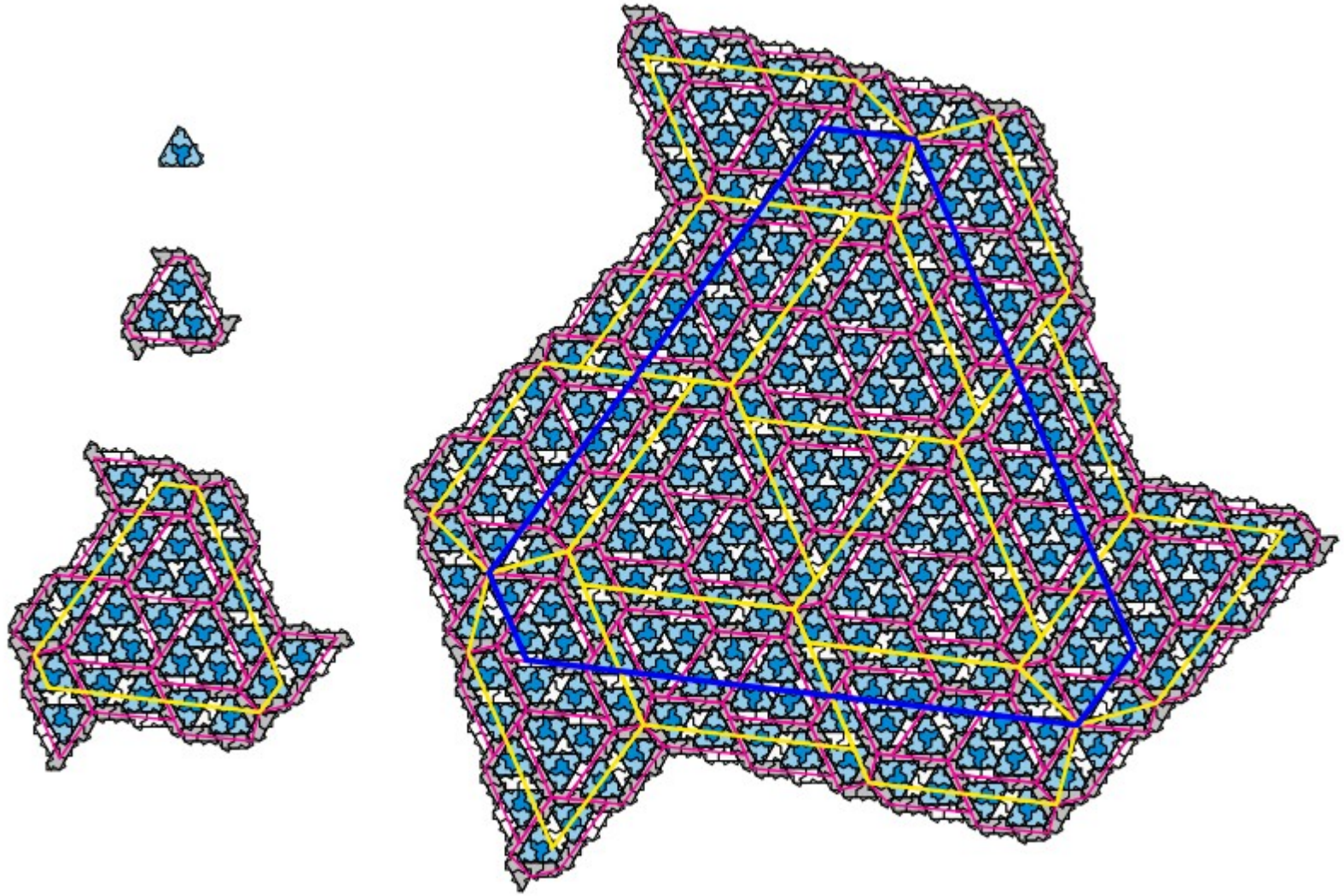
- Paver l’espace de meta-tiles
- Trouver des “clusters”
- Remplacer les clusters par des “meta-meta-tiles”



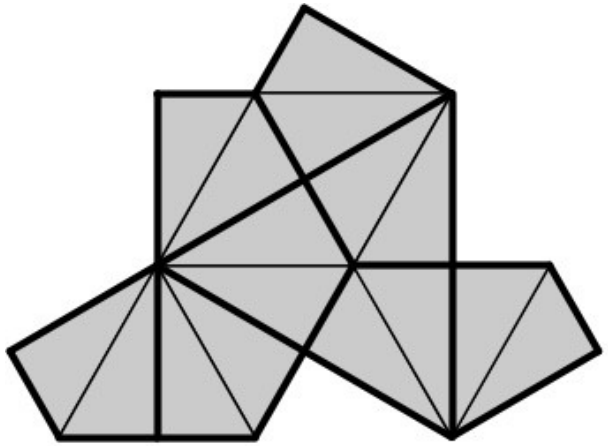




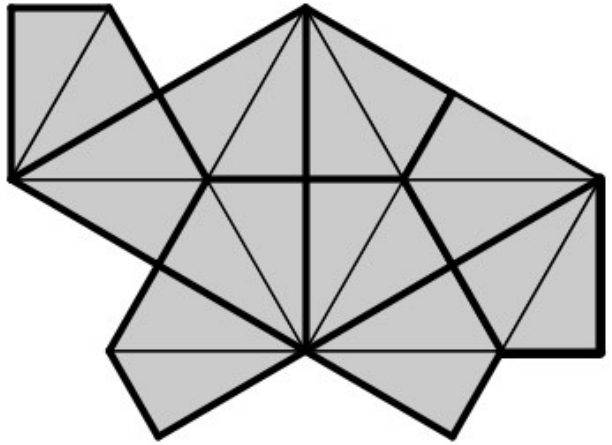
Apériodique!



La “Tortue”

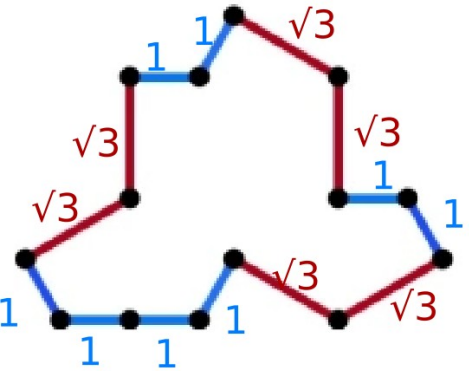


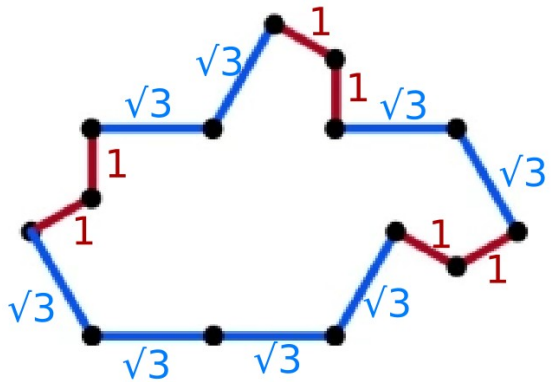
hat

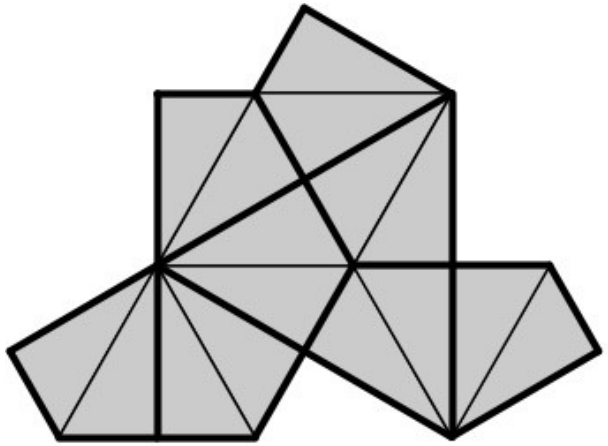


turtle

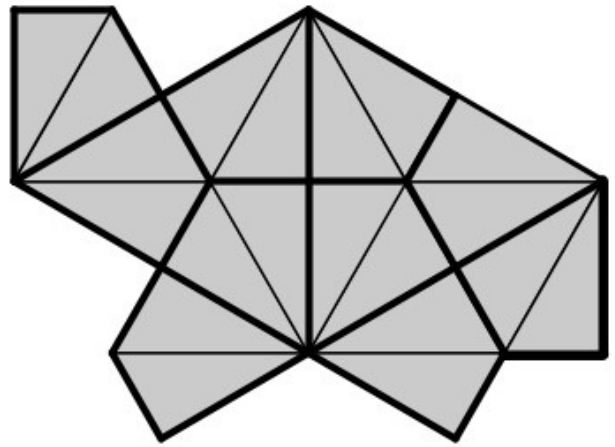
La “Tortue”





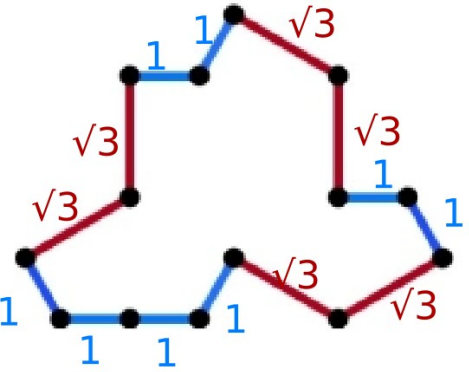


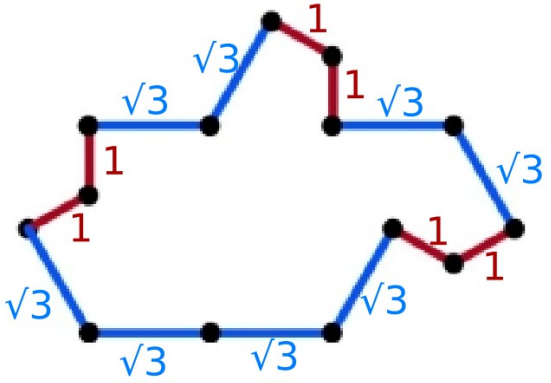
hat

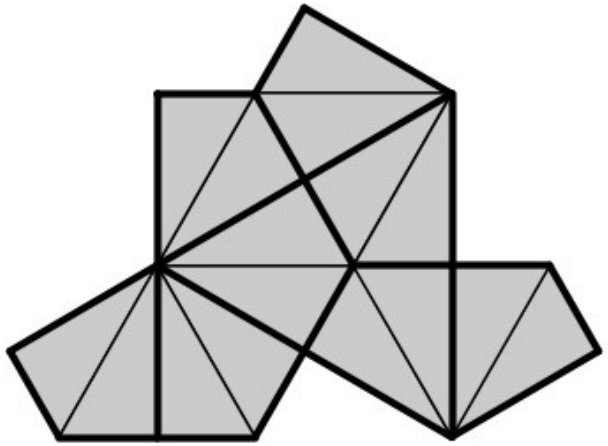


turtle

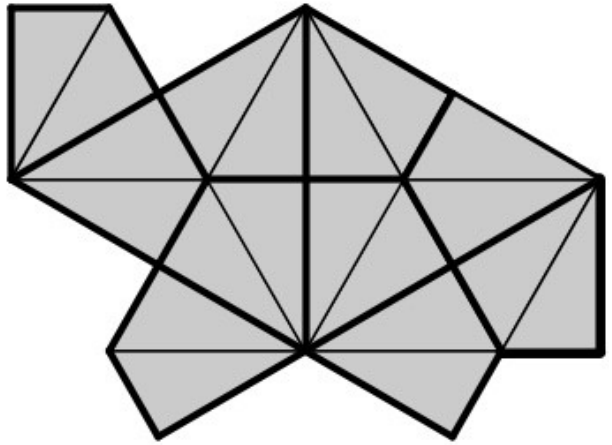
La “Tortue”



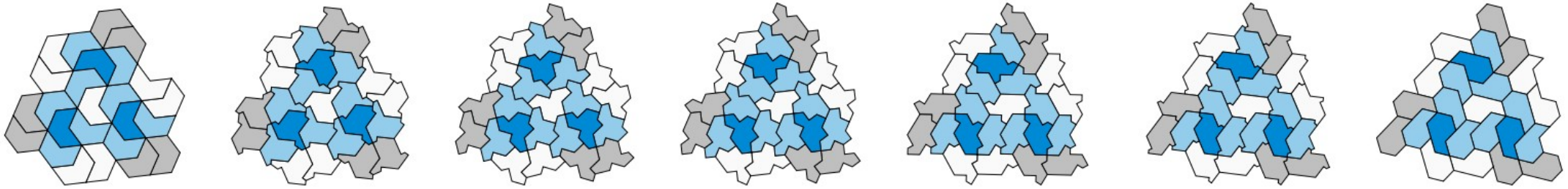




hat

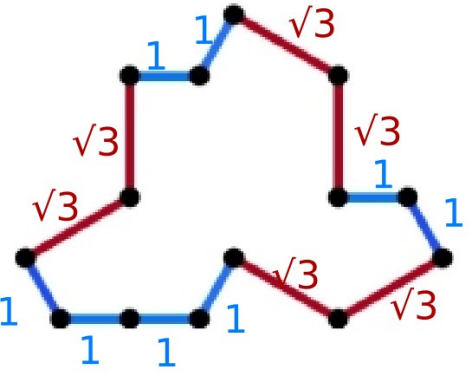


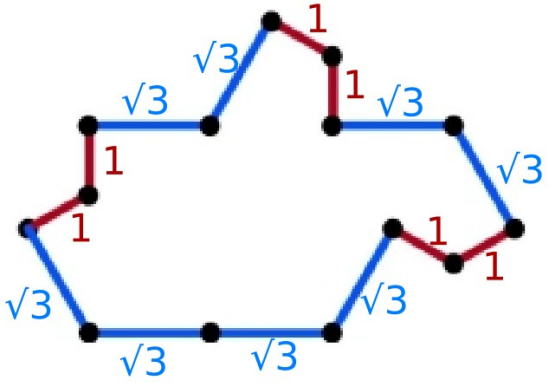
turtle

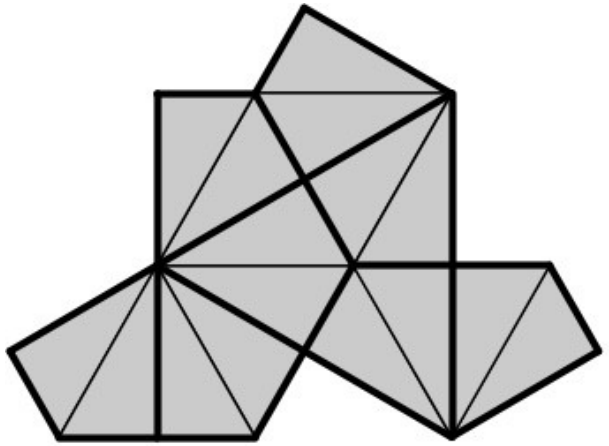


$\mathrm{Tile}(0,1)$ $\mathrm{Tile}(1,4)$ $\mathrm{Tile}(1,\sqrt{3})$ $\mathrm{Tile}(1,1)$ $\mathrm{Tile}(\sqrt{3},1)$ $\mathrm{Tile}(4,1)$ $\mathrm{Tile}(1,0)$

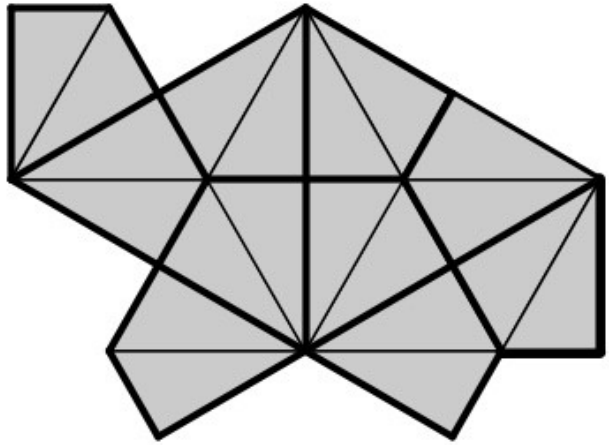
La “Tortue”



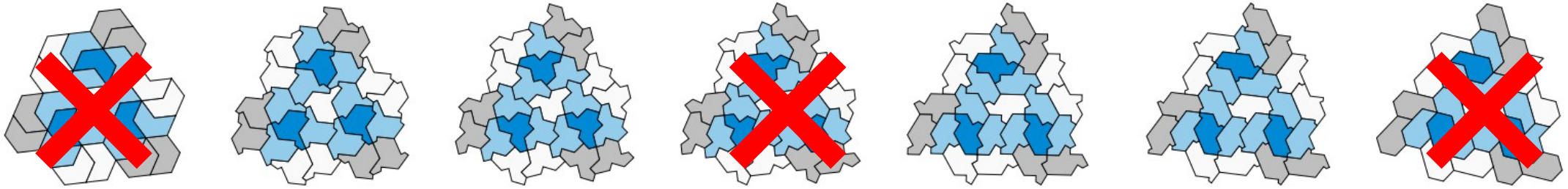




hat

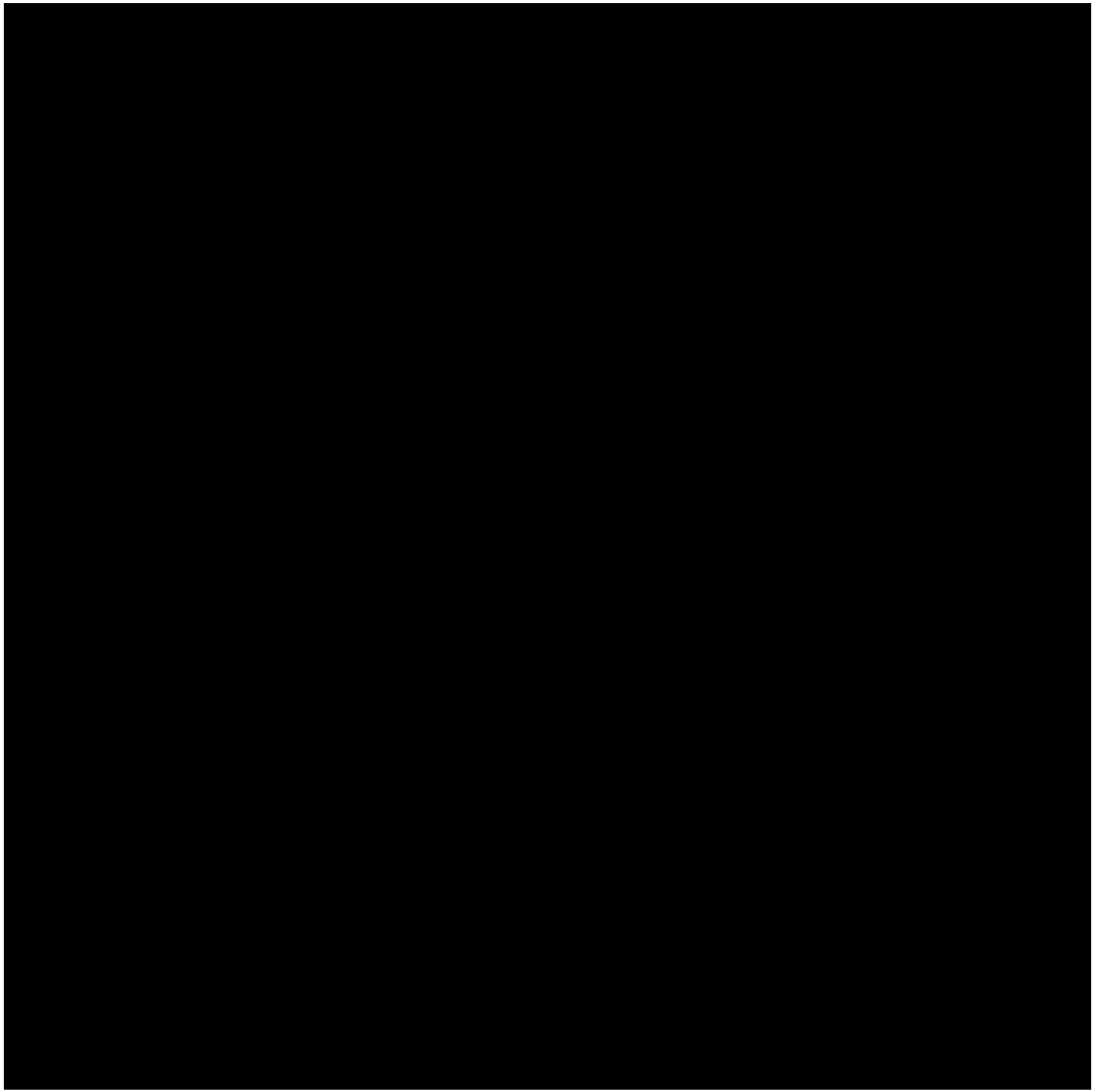


turtle



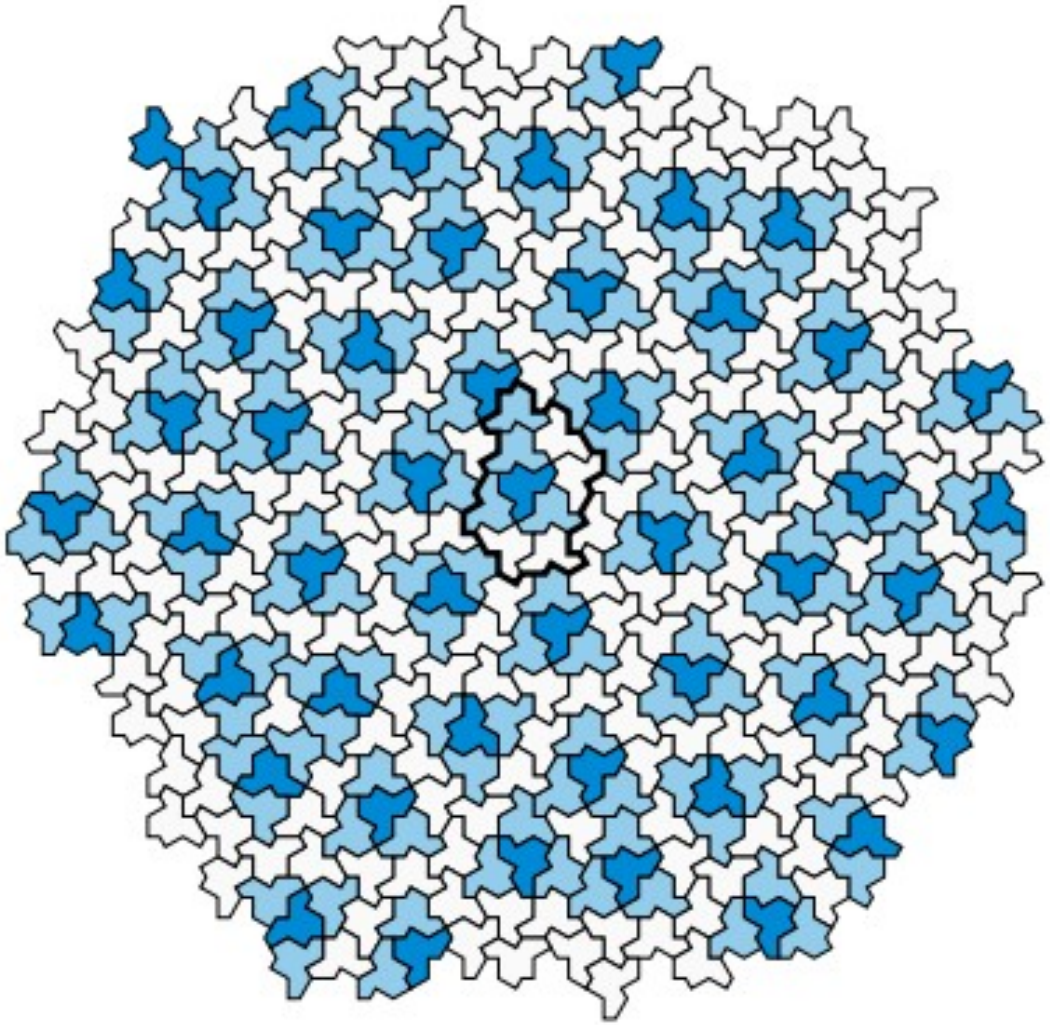
$\mathrm{Tile}(0,1)$ $\mathrm{Tile}(1,4)$ $\mathrm{Tile}(1,\sqrt{3})$ $\mathrm{Tile}(1,1)$ $\mathrm{Tile}(\sqrt{3},1)$ $\mathrm{Tile}(4,1)$ $\mathrm{Tile}(1,0)$

La “Tortue”



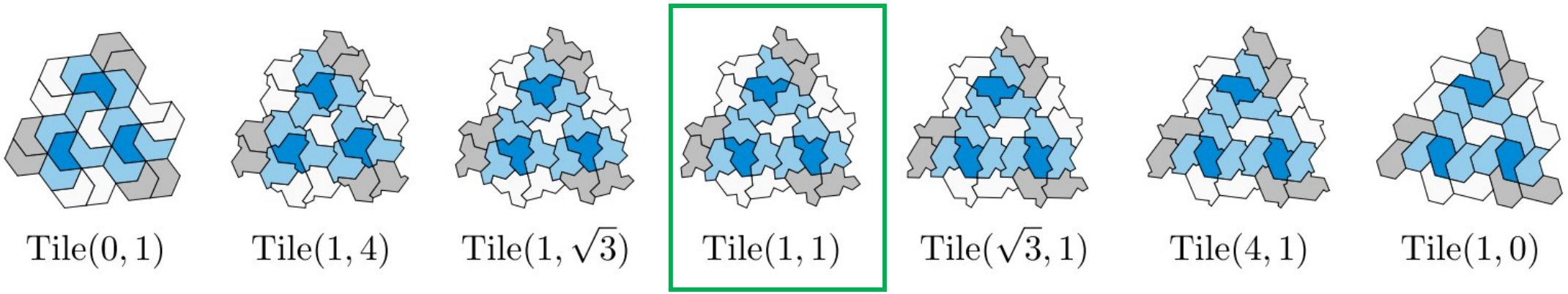
Les haters et la chiralité

- Utilisation de “mirroirs” dans nos pavages (bleu foncé)
- = 2 tuiles?



Le Spectre, le seul, l'unique...

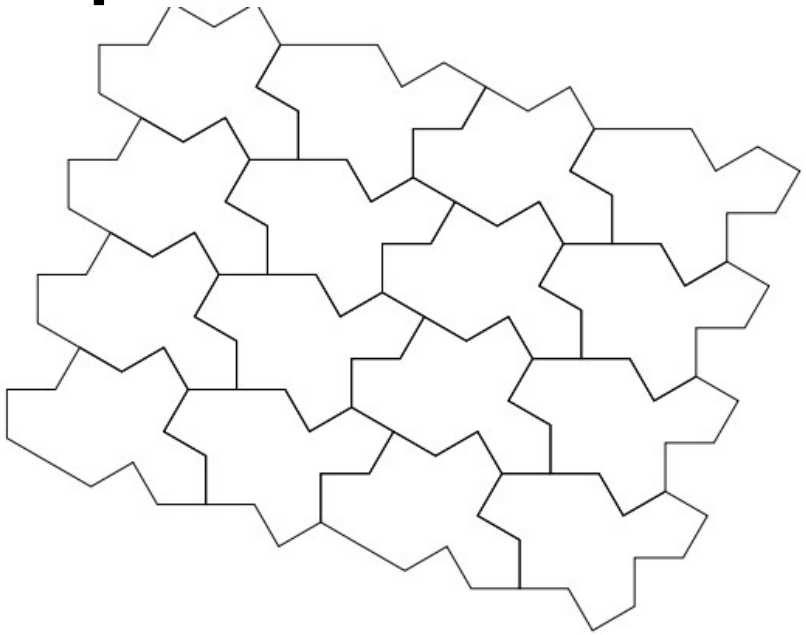
- Utilisation de la tuile(1, 1)
- Elle est non-apériodique...

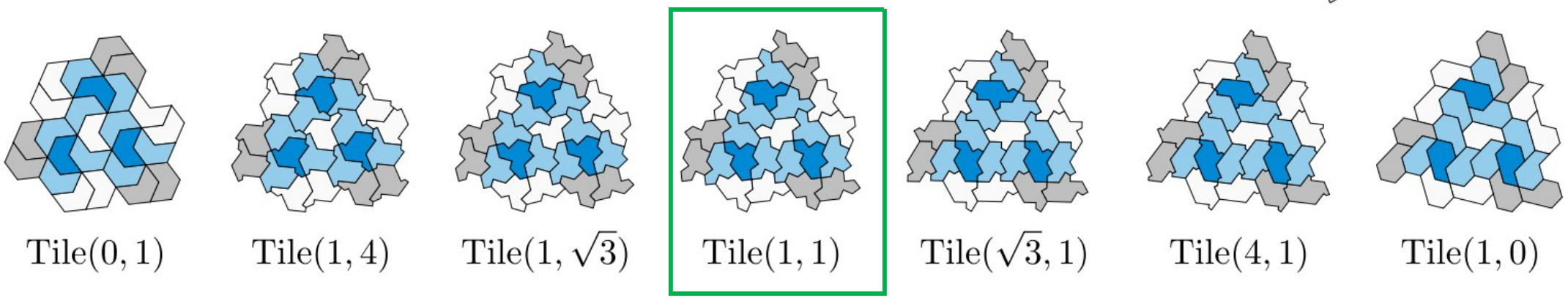


Smith, Myers, Kaplan and Goodman-Strauss, *A chiral aperiodic monotile* (2023)

Le Spectre, le seul, l'unique...

- Utilisation de la tuile(1, 1)
- Elle est non-apériodique...

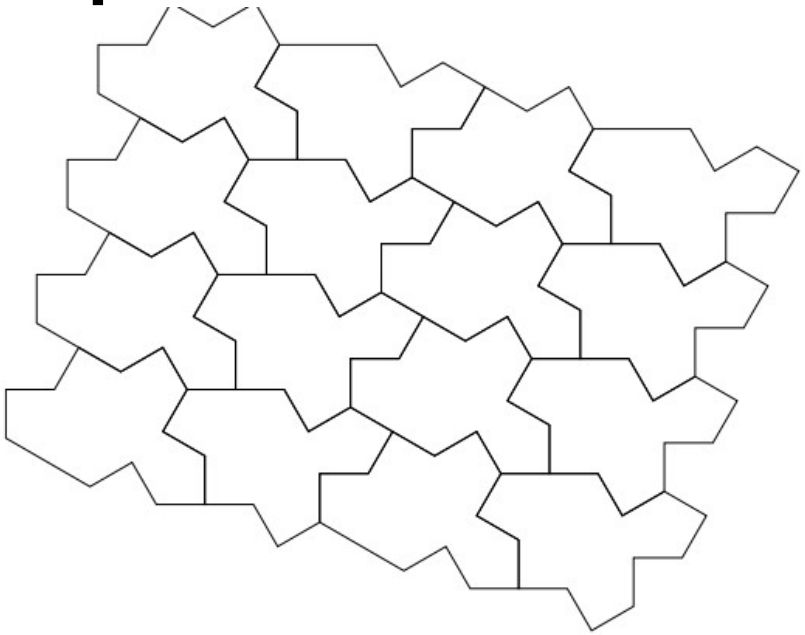


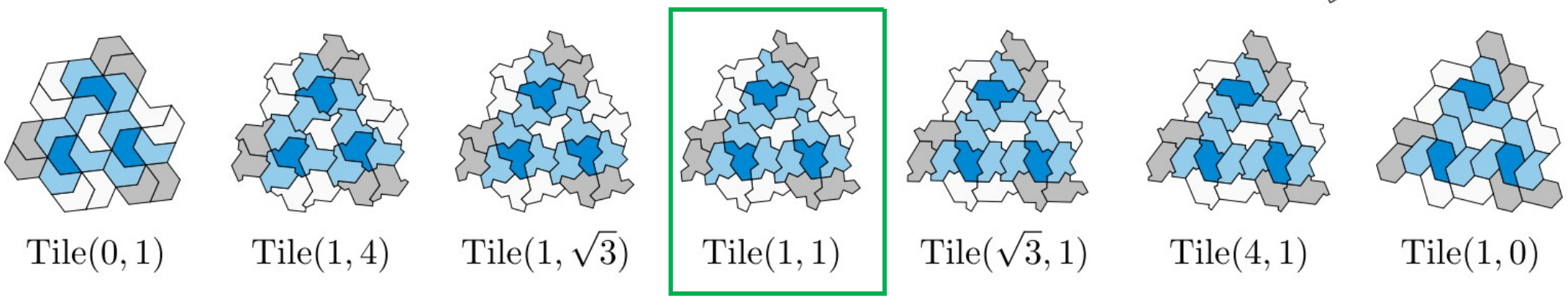


Smith, Myers, Kaplan and Goodman-Strauss, *A chiral aperiodic monotile* (2023)

Le Spectre, le seul, l'unique...

- Utilisation de la tuile(1, 1)
- Elle est non-apériodique...
- ... si on autorise son mirroir

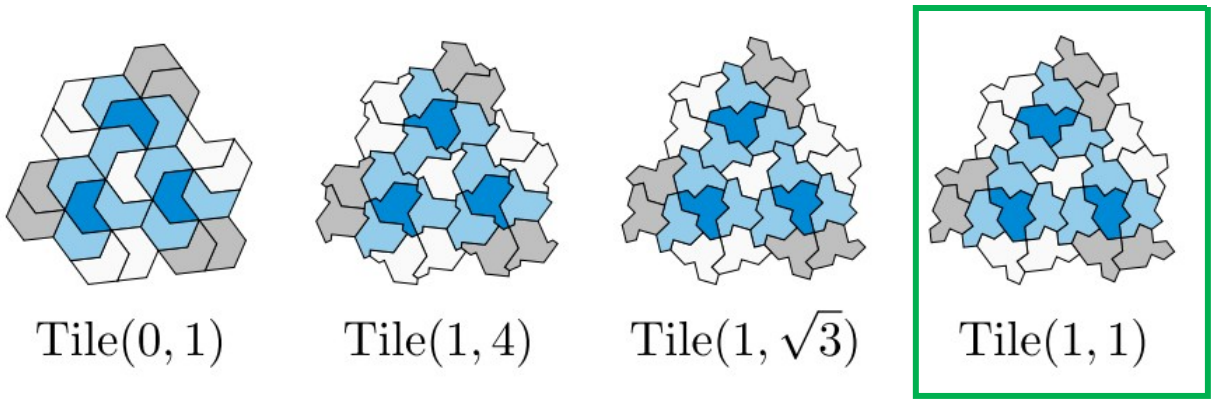


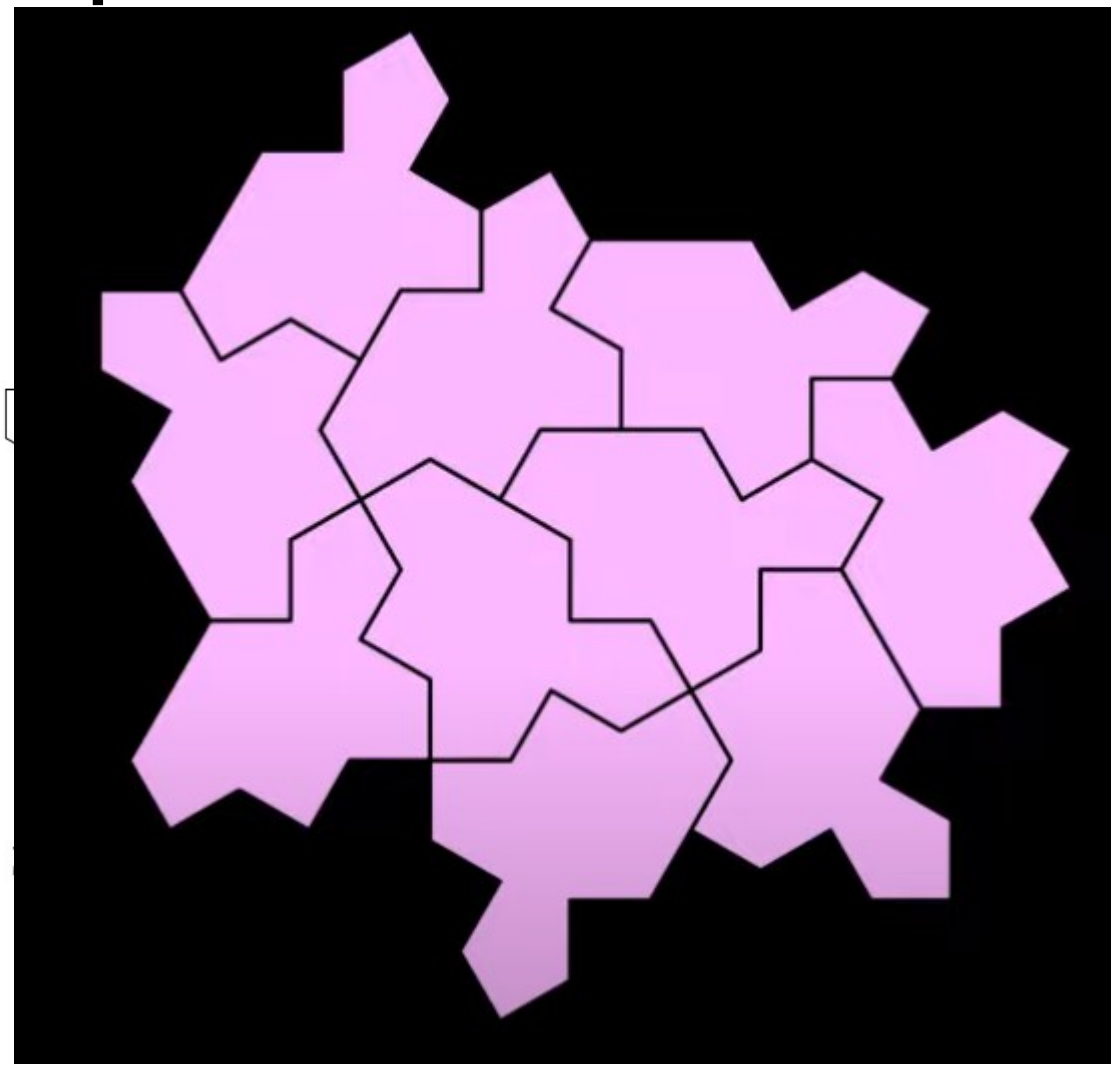


Smith, Myers, Kaplan and Goodman-Strauss, *A chiral aperiodic monotile* (2023)

Le Spectre, le seul, l'unique...

- Utilisation de la tuile(1, 1)
- Elle est non-apériodique...
- ... si on autorise son mirroir

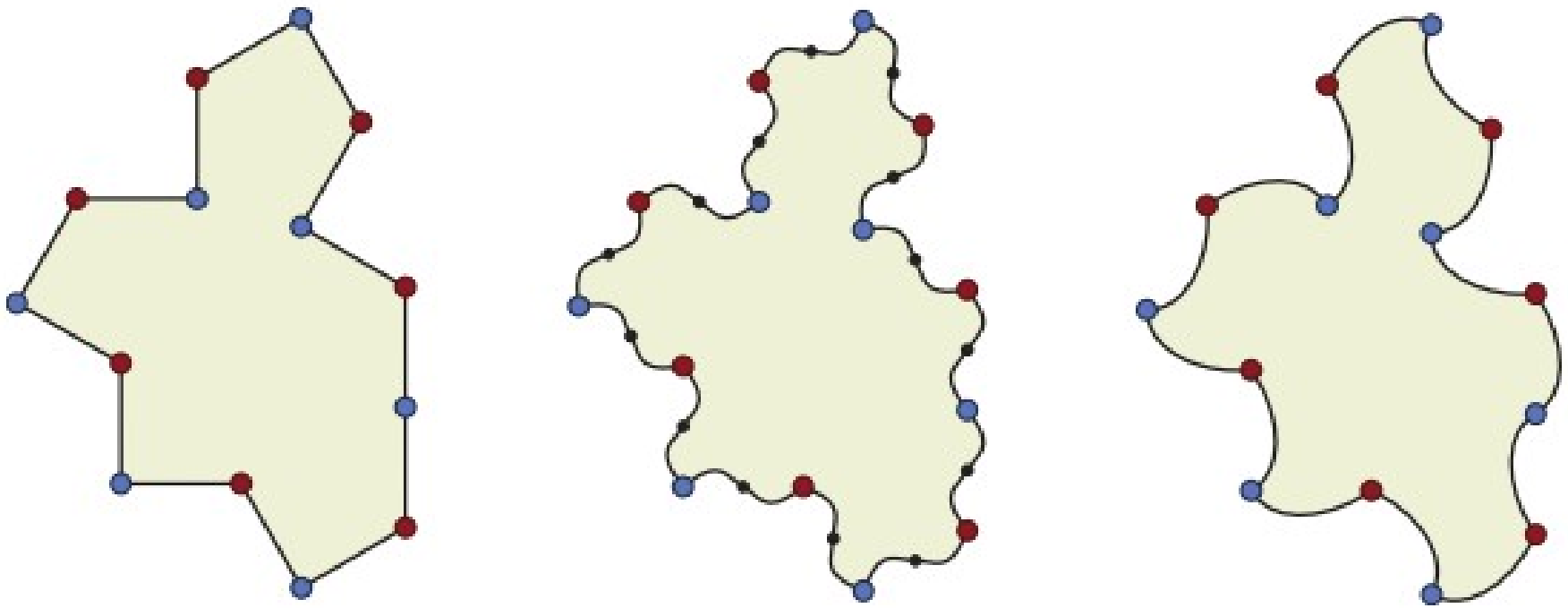




Smith, Myers, Kaplan and Goodman-Strauss, *A chiral aperiodic monotile* (2023)

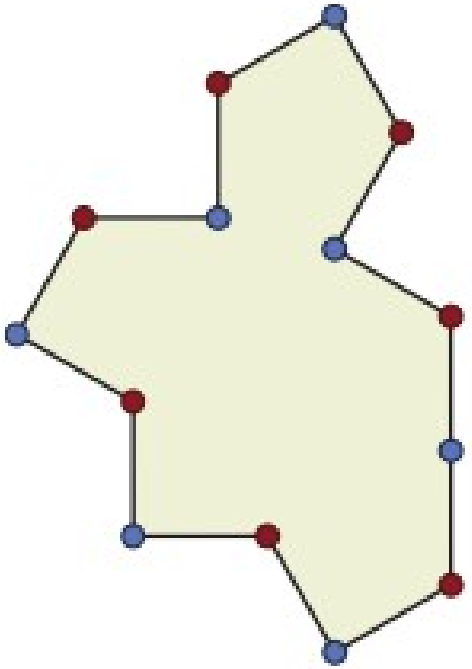
Le Spectre, ni seul, ni unique!

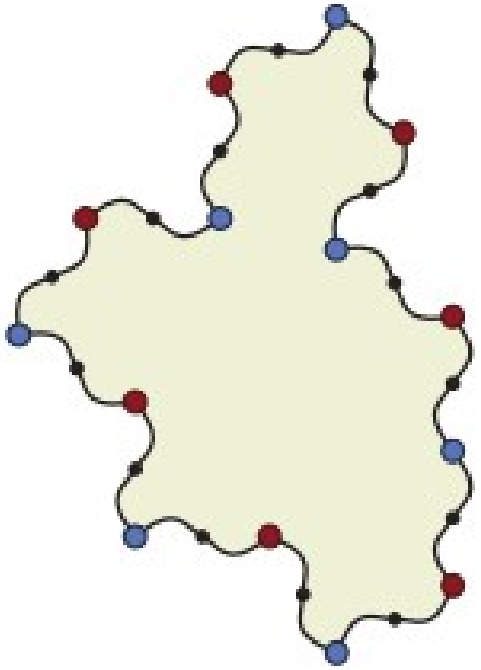
- Ajouter des courbes, et bam!

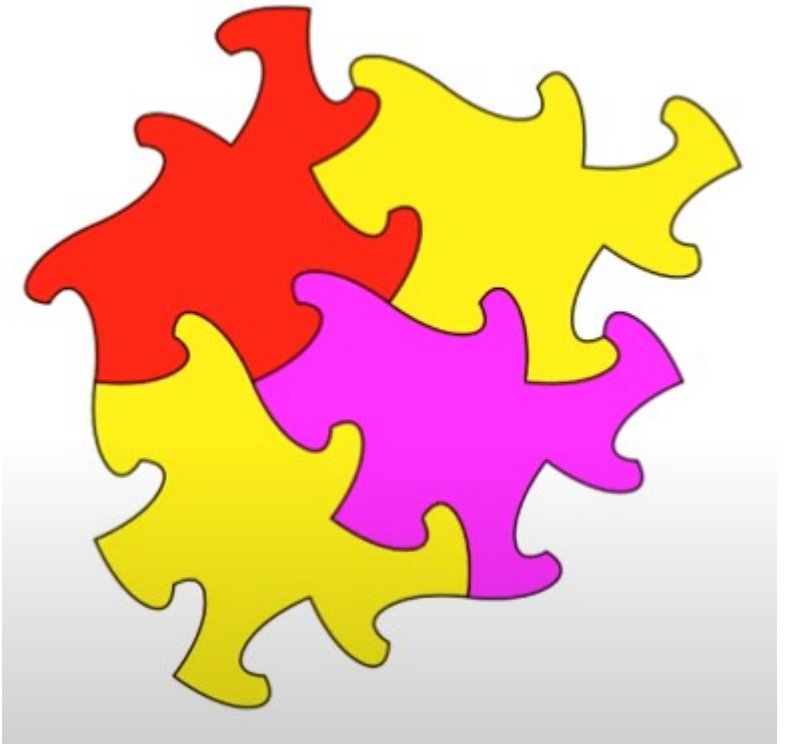


Le Spectre, ni seul, ni unique!

- Ajouter des courbes, et bam!





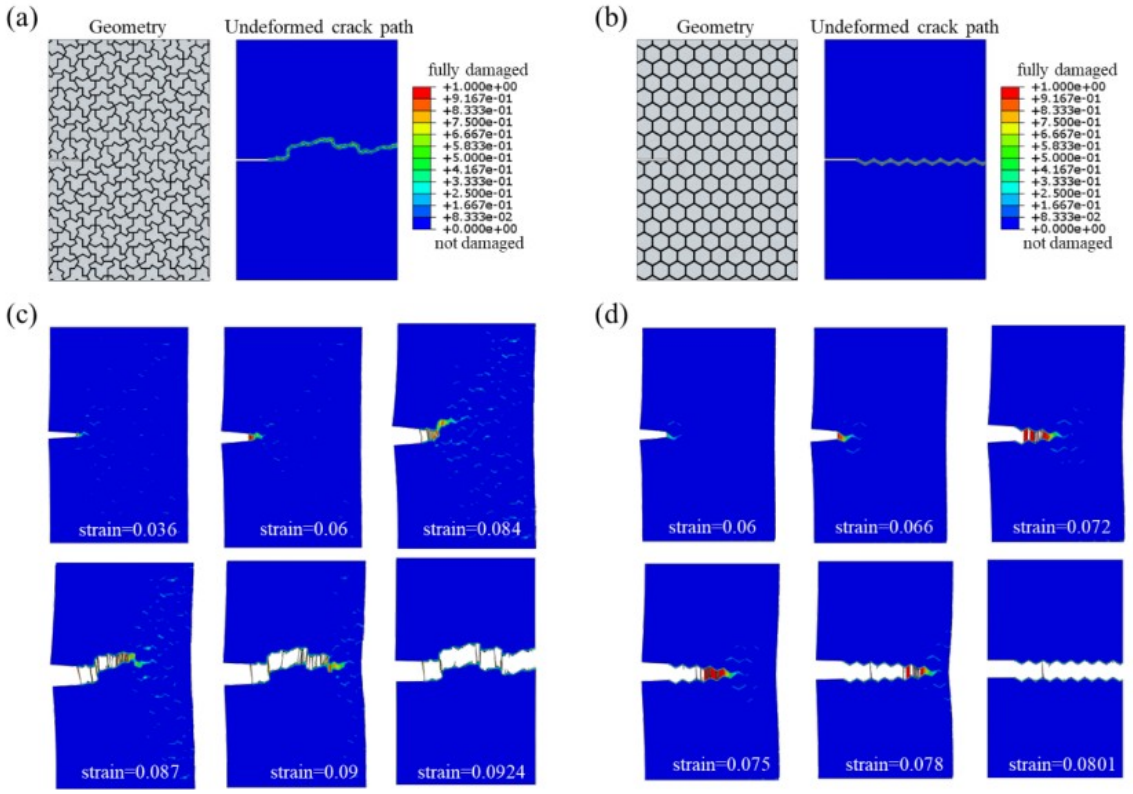


Conclusion

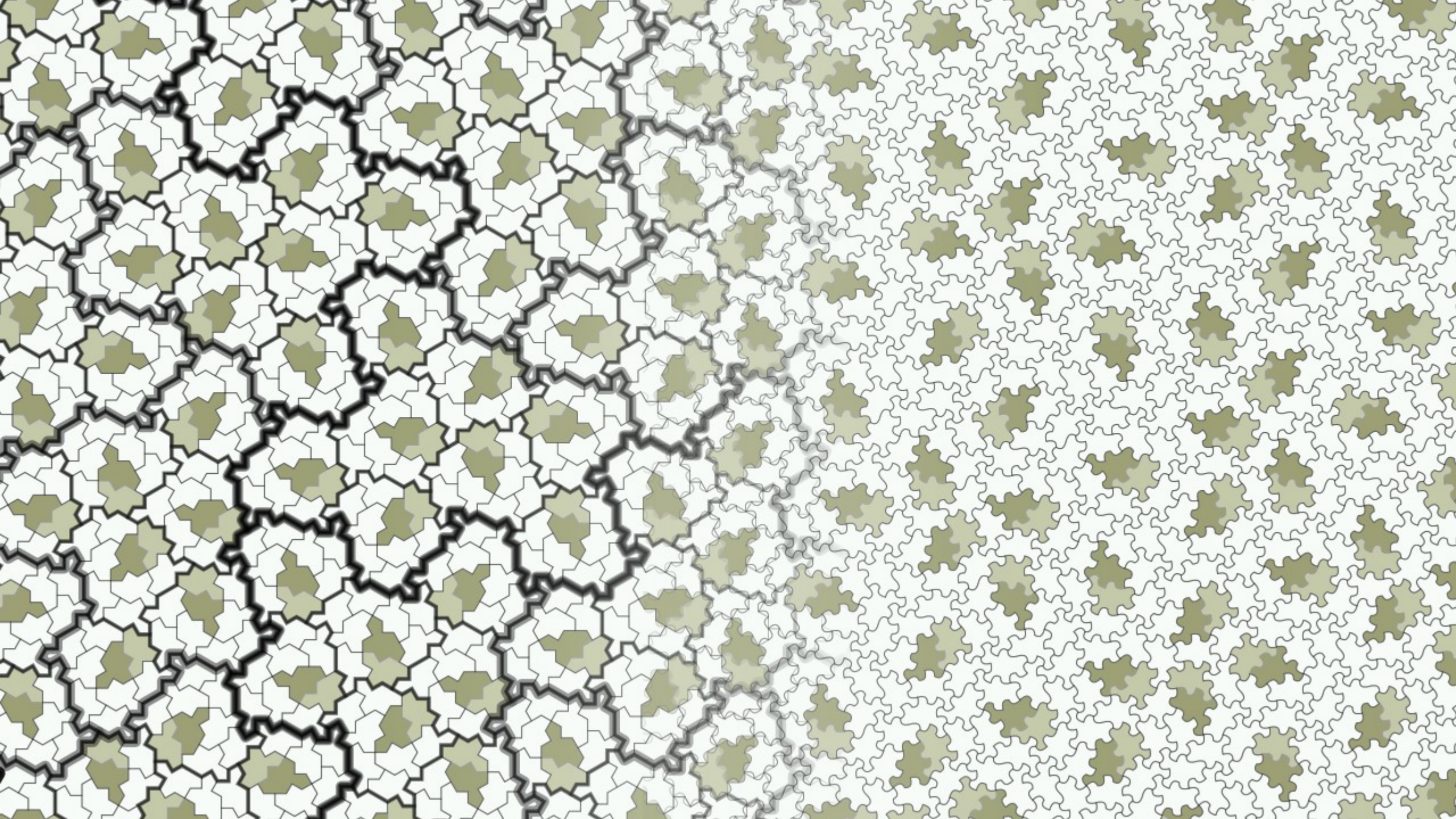
- Un problème gelé depuis 50 ans débloqué
- Découverte d’une famille continue de tuiles apériodiques avec problème de symmétrie
- Découverte d’une famille continue de tuiles apériodiques sans problème de symmétrie

Et après..?

- Des recherches sur la solidité de matériaux, impression 3D
- Bah... C’est tout!



Jung, Chen and Gu, *Aperiodicity is all you need: Aperiodic monotiles for high-performance composites* (2024)



Remerciements

Numberphile

@numberphile · 4,49 M d'abonnés · 724 vidéos

Videos about numbers and mathematics. Videos by Brady Haran since 2011. **...plus**

numberphile.com **et 3 autres liens**

Abonné

Combo Class

@ComboClass · 40 k abonnés · 64 vidéos

Hey, I'm Domotro. Welcome to Combo Class! Get ready to learn some crazy things about **...plus**

youtube.com/channel/UC2RpxrkilRbhwED1YG4BUIA **et 4 autres liens**

Abonné

Aylean

@Aylean · 10,6 k abonnés · 74 vidéos

I just love maths and I need a place to nerd out. **...plus**

S'abonner

Mostly Mental

@mostly_mental · 1,88 k abonnés · 30 vidéos

En savoir plus sur cette chaîne **...plus**

S'abonner

Up and Atom

@upandatom · 726 k abonnés · 125 vidéos

Making hard stuff less hard. **...plus**

patreon.com/upandatom **et 1 autre lien**

S'abonner

Mickaël Launay

@Micmaths · 579 k abonnés · 196 vidéos

La chaîne Micmaths vous propose diverses vidéos autour des mathématiques. Vous y tro **...plus**

mamot.fr/@mickaellaunay **et 4 autres liens**

Abonné

Mathologer

@Mathologer · 927 k abonnés · 103 vidéos

Enter the world of the Mathologer for really accessible explanations of hard and beautiful **...plus**

patreon.com/mathologer **et 4 autres liens**

Abonné